

# Timely detection of turning points: Should I use the seasonally adjusted or trend estimates?

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Timely detection of turning points: Should I use the seasonally adjusted or trend estimates?

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## 1. Introduction

The timely and accurate detection of turning points is an important issue in analysing time series data. Different time series estimates, such as the original estimates and the derived seasonally adjusted and trend estimates, are available to help assess turning points. Auxiliary information can also be used. The Australian Bureau of Statistics (ABS) regularly publishes original, seasonally adjusted and trend estimates to enable users a choice of complimentary time series estimates. Users may choose to use any, or all of, the time series estimates as provided or as input into sophisticated modelling approaches which can then assist with informed judgement, decision and policy making. The ABS recommends the use of trend estimates to provide the most appropriate estimate of the underlying direction of the original time series (Linacre and Zarb, 1991). Knowles (1997) and Compton (2000) surveyed trend estimation practice of a range of National Statistical Institutes and found that quick detection of turning points and minimisation of the number of false turning points were desirable characteristics of short-term trends. Knowles and Kenny (1997) considered issues with turning points for trend estimates. If a turning point is incorrectly identified or not identified soon enough, it may lead to inaccurate assessments of economic activity, which may in turn impact on important economic decisions. Seasonally adjusted estimates for official government statistics are typically derived using a univariate approach for individual time series. Alternative multivariate approaches which use relationships between time series can improve the detection of turning points (for example, see Zhang and McLaren, 2005). This paper focuses on detection of turning points from time series estimates derived using a univariate approach.

We consider issues in detecting turning points for monthly time series in using either the published seasonally adjusted and trend estimates available from a filter based seasonal adjustment process. The trend estimate is often perceived to be a signal extraction or data transformation process and the use of filters are known to introduce distortion. We investigate if there is a trade-off between the fast detection of turning points and the risk of false positive detection of turning points. We examine the factors influencing the detection of turning points. This work is ongoing and the purpose of this paper is to stimulate comment and debate. Additional updated information is now available in ABS (2006c).

## 2. Background

Assume a multiplicative decomposition model for the original estimates at time  $t$ ,  $O_t$ , based on a filter based seasonal adjustment approach. For example, see X-12-ARIMA (Findley et. al, 1998). This can be written as a combination of the combined seasonal factor  $S_t$ , the trend  $T_t$ , and the irregular  $I_t$ ,

$$O_t = T_t \times S_t \times I_t$$

The seasonally adjusted estimates ( $SA_t$ ) are derived from the original estimates by estimating and removing the systematic calendar related component:

$$SA_t = O_t / \hat{S}_t = T_t \times I_t$$

The seasonally adjusted estimates contain both the trend and irregular components. Movements in the seasonally adjusted estimates will be influenced by the irregular component which can mask the underlying direction of the series. The trend estimates ( $T_t$ ) are derived from the seasonally adjusted estimates by smoothing the irregular component, for example, by using the Henderson filter (Henderson, 1916). The trend estimate is an attempt to represent the underlying direction of a time series which is influenced by general changes in the economy such as population growth. Trend estimates are smoother and show gradual movements when compared to seasonally adjusted estimates. The Henderson definition of the trend estimate includes cycles of approximately eight months or greater which would include the business cycle. Alternative definitions can extract the business cycle as a separate component. The definition of trend estimates is a non-trivial issue and can depend on the need of individual users.

## 3. Defining a turning point

Typically, a turning point is defined by a monotonically decrease (increase) sequence followed by an increase (decrease) sequence over a given number of time periods. Knowles and Kenny (1997) use this definition for an upturn,

$$Y_{t-k} > Y_{t-k+1} > \dots > Y_t, \quad Y_t < Y_{t+1} < \dots < Y_{t+m} \quad (1)$$

A downturn can be similarly defined. Values for  $k$  and  $m$  can be varied. See for example, Wecker (1979). The earliest a turning point can be determined at the current end is after an additional  $m$  periods are available. Equation (1) requires the series to be monotonically increasing or decreasing on either side of the turning point. By definition, the seasonally adjusted estimates contain the irregular component which can mean that the period to period movements can vary considerably between consecutive time periods. Intuitively this means that turning points in the seasonally adjusted estimates may not be found using this definition. Equation (1) also does not take into account the magnitude of the movements in the level of the series. Alternative definitions for turning points can be considered by relaxing the strict monotonic constraint (eg. Bry and Boschan, 1971). An example of an alternative definition is the Phase-Average-Trend approach (Watson, 1994). The Phase-Average-Trend approach is not strongly biased to either the trend or seasonally adjusted estimates so it will be more appropriate for comparison purposes.

This paper currently considers (1) with appropriate choice of  $k$  and  $m$ . Selected choices for  $k$  and  $m$  are discussed in Section 5. Alternative, more appropriate, definitions for a turning point have been considered in ABS (2006c).

#### **4. Comparing turning point detection for seasonally adjusted and trend estimates**

We consider both real and simulated time series and the derived seasonally adjusted and trend estimates. Turning points were calculated for individual time series at each consecutive time point and compared against the “true” turning points (benchmark). We evaluate the properties of the detected turning points, such as, the number of false turning points where the detected turning point did not match the benchmark, and the timeliness of detection of turning points with the length of elapsed time to detect a turning point in the benchmark series.

##### **4.1 Comparison against the benchmark estimates**

A benchmark series defines the “true” turning points and is needed to determine if a detected turning point in a given time series at a particular time period corresponds to the “true” turning point. Ideally, the benchmark should be determined independently of the process used to estimate the turning points. We would like to minimise bias in either the seasonally adjusted or trend estimates when comparing against the benchmark in order to make a fair comparison. For example, by using the same type of time series estimate for both detection and the calculation of the benchmark. A benchmark series is calculated by using the full length original time series and deriving the seasonally adjusted and trend estimates. Turning points are determined along the length of the time series by adding original estimates and calculating the seasonally adjusted and trend estimates each time and then comparing this against the respective benchmark series. Additional original estimates were added until three years from the end of the full length time series. The seasonally adjusted and trend estimates will be revised as additional original estimates are added which can result in turning points being lost and found.

##### **4.2 Quantifying differences between seasonally adjusted and trend**

We considered the following issues for both the seasonally adjusted and trend estimates:

- a. Determining an arbitrary choice of  $k$  and  $m$  in the definition of a turning point (1) for seasonally adjusted and trend estimates.
- b. Timeliness until a turning point is detected should be minimised and is assessed for both the seasonally adjusted and trend estimates. Timeliness is defined to be the average number of periods before a turning point is first detected.

c. False turning points. A false turning point is defined to be a time point that is detected as a turning point in a time series but does not exist as a turning point in the corresponding benchmark series. The proportion of false turning points is the average proportion of turning points in the most recent time period which do not match with turning points detected in the benchmark series. For example, if  $k=m=3$  and data for December is available for a monthly series, the proportion of false turning points is the average proportion of turning points occurring between April and September (inclusive) that do not match with turning points detected in the benchmark series.

## 5. Results

Results were calculated using monthly real and simulated time series. Different realisations of monthly time series,  $Y_t$ , were simulated using the airline model (Box and Jenkins, 1976),  $(1-B)(1-B^{12})Y_t=(1-\theta B)(1-\Theta B^{12})\varepsilon_t$  where  $B$  is the backshift operator,  $\varepsilon_t$  is a white noise process and parameters  $\theta = 0.5$  and  $\Theta = 0.7$ . The volatility of the input white noise process,  $\varepsilon_t$ , was controlled. The airline model with appropriate parameter choice adequately fits a large proportion of ABS time series (approximately 80%, ABS, 2001). Seasonally adjusted and trend estimates were then calculated using a derivation of the X-12-ARIMA method. Real data used was the Total Short Term Visitor Arrivals (ABS, 2006a) and Total Employed Persons and Total Unemployed Persons (ABS, 2006b). All turning points were defined using equation (1).

### 5.1 Appropriate values for $k$ and $m$ in defining a turning point

Figure 1a shows the relationship between values for  $k$  and  $m$ , where  $k=m$ , in (1) for 59 simulated time series and the mean number of detected turning points calculated using the full span of data (benchmark). The choice of  $k$  and  $m$  is arbitrary, but there needs to be a balance in the number of turning points identified. In general, more turning points are detected for smaller values of  $k$  and  $m$ . Incrementing  $k$  and  $m$  results in a gradual decline in the number of turning points detected in the trend series, but a rapid decline in the number of turning points detected in the seasonally adjusted series. Values for  $k$  and  $m$  of greater than four reduce the number of turning points detected in the seasonally adjusted series to zero due to the contribution of the irregular component. Small values for  $k$  and  $m$  give an increased number of detected turning points. Figure 1b shows a similar relationship between values for  $k$  and  $m$  and the mean number of turning points detected using three real time series (Total Australian Employed Persons, Total Australian Unemployed Persons, Short Term Visitor Arrivals). We use  $k=m=3$  for this study as this gives reasonable turning point detection for both the seasonally adjusted and trend estimates. Alternative choices could be used.

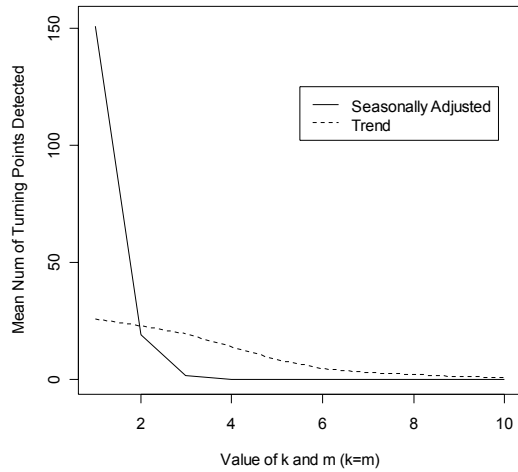


Figure 1a: Sensitivity of the number of turning points to  $k$  and  $m$  ( $k=m$ ) for simulated time series.

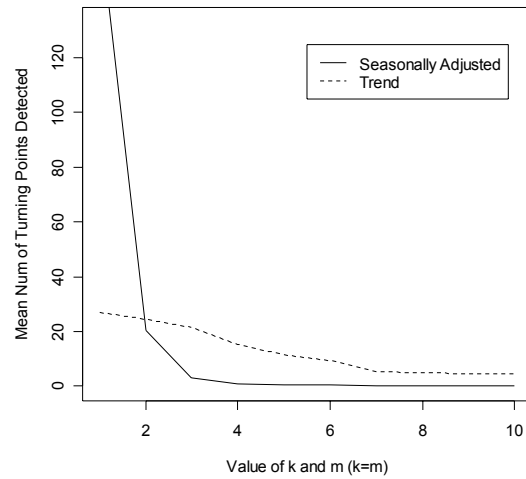


Figure 1b: Sensitivity of the number of turning points to  $k$  and  $m$  ( $k=m$ ) for selected real time series.

## 5.2 Timeliness of turning point detection

We compare seasonally adjusted and trend estimates for detection against different benchmarks using both real and simulated (monthly) example series (where  $k=m=3$ ).

Figure 2a shows the proportion of turning points detected in the Employed Persons series relative to the months after the occurrence of the turning points. Using the trend as the benchmark and to also detect turning points (T,T) results in an average time to detection of 13 months with all turning points detected within 36 months. Using the seasonally adjusted series for both the benchmark and for detection (SA,SA) gives an average detection time of 9 months with all turning points detected within 16 months, indicating earlier detection in the seasonally adjusted series. However, it is 7 months before any turning points are detected in the seasonally adjusted series when over half the turning points have been detected in the trend series. If trend estimates are used for the benchmark and seasonally adjusted estimates for detection (and vice versa, T,SA and SA,T) the proportion of turning points detected is low and unstable.

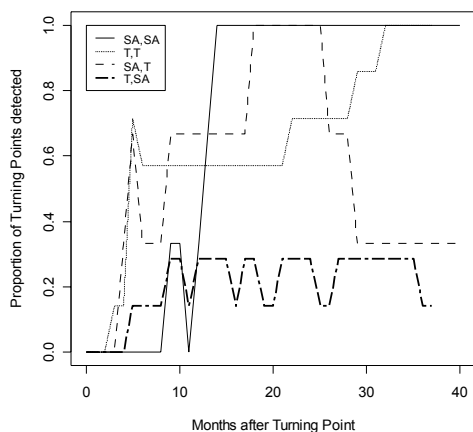


Figure 2a. Timeliness of turning point detection against the proportion of Turning Points Detected in Employed Persons: Notation SA,T uses the trend for the benchmark and seasonally adjusted for detection.

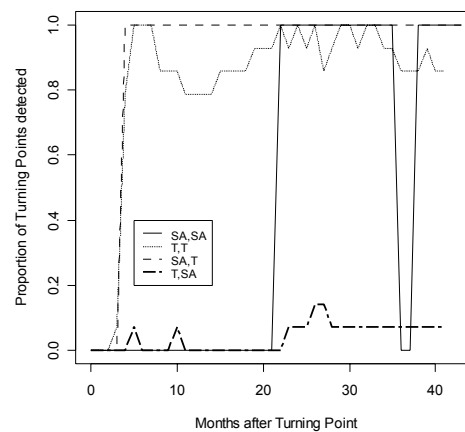


Figure 2b. Timeliness of turning point detection against the proportion of Turning Points Detected in Unemployed Persons: Notation SA,T uses trend for the benchmark and seasonally adjusted for detection.

Figure 2b shows the Unemployed Persons time series which has a small number of turning points in the seasonally adjusted estimates and shows quite different behaviour. The trend estimate quickly detects the one turning point in the seasonally adjusted benchmark within 5 months. The seasonally adjusted estimate does not detect the turning point in the seasonally adjusted benchmark until after 20 months, and loses it again for several months approximately 3 years later. The seasonally adjusted series performs poorly in the detection of the trend benchmark turning points. Detection of trend benchmark turning points by the trend estimates is somewhat affected by the instability in the trend.

The majority of the turning points are detected quickly. The above comparison highlights the need for a more robust turning point criteria suitable for both seasonally adjusted and trend estimates in comparing against the benchmark turning points for a fair comparison.

Using simulated series, seasonally adjusted and trend estimates are compared against the respective seasonally adjusted and trend benchmarks. Figure 3a shows the average number of months before turning points are first detected using seasonally adjusted estimates which can be up to 60 months. For some time series, the average time can be between 65 and 80 months. Figure 3b shows that for trend estimates the majority of the series take between 10 and 15 months to detect turning points. The maximum average time to detect turning points is 30 months. This is significantly less than using the seasonally adjusted estimates.

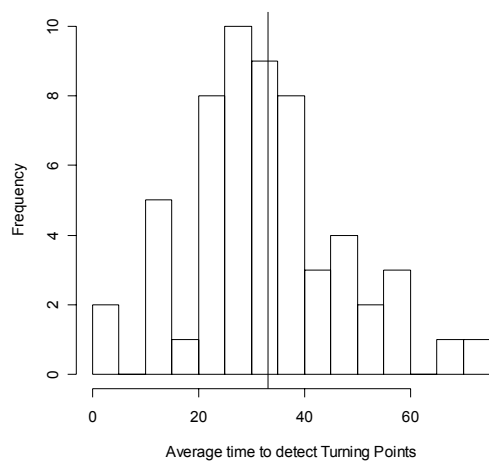


Figure 3a. Seasonally adjusted: Average months to detect turning points in simulated time series.

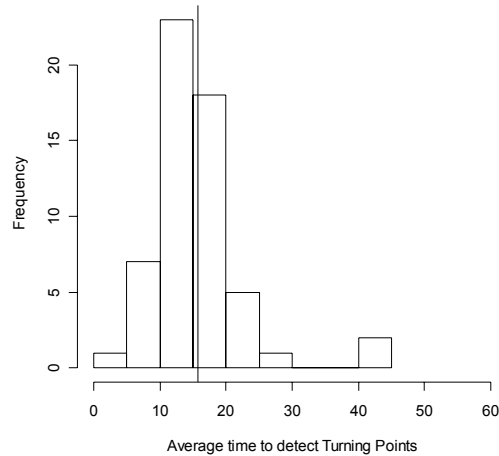


Figure 3b. Trend: Average months to detect turning points in simulated time series.



### 5.3 False and true detection of turning points

It is desirable to minimise how often a time point is incorrectly detected as a turning point. We have compared the same type of estimate (seasonally adjusted and trend) for both detection and the benchmark. This analysis is useful to gain an understanding of how each time series estimate performs. Comparison between the seasonally adjusted and trend estimates should be considered carefully as the benchmark estimates are different. Table 1 shows that the proportion of false turning points in the seasonally adjusted estimates was found to be much higher than in the trend estimates for real data.

This is due to the greater level of irregularity present in the seasonally adjusted estimates. For example, consider Employed Persons. For this series, 18.2% of turning points detected in the last 6 months are false when using the trend but almost 70.0% if using the seasonally adjusted series. For Short Term Arrivals to Australia, the difference between the seasonally adjusted and trend estimates is less extreme. In the last six months, 28.7% of turning points detected using trend series were false as compared to 40.0% in the seasonally adjusted series. In addition, only 16.0% of turning points occurring 12-24 months ago in the trend series were false, whilst over 36.6% in the seasonally adjusted were false. A similar pattern was observed for the Unemployed Persons.

In the seasonally adjusted series, the revisions to the seasonally adjusted estimates as new original estimates are added tend to be small but continue for many months. In the trend series, the revisions initially occur for several months before becoming very gradual. False positives tend to be more prevalent and remain longer in the seasonally adjusted series than in the trend series.

		Simulated (mean of 59 series)	Employed Persons	Unemployed Persons	Short Visitor Arrivals	Term
Total number of turning points (up and down)		2.45	4		1	4
	T	19.0	10		22	32
Average detection time <sup>^</sup> (exact)	SA	33.1	9.3		19.0	3.0
	T	15.8	13.0		13.5	10.0
Average detection <sup>*^</sup> time (inexact)	SA	33.1	9.0		19.0	3.0
	T	15.8	5.0		1.1	5.9
% False positives* <6 months	SA	14.0	70.0		100.0	40.0
	T	75.4	18.2		19.6	28.7
% False positives* 6-12 months		17.0	55.2		100.0	26.1
	T	71.0	21.1		19.4	18.7
% False positives* 12-24 months	SA	24.0	32.7		33.3	36.6
	T	63.5	18.1		4.2	16.0

Table 1. Comparison of trend estimates to trend benchmark and seasonally adjusted estimates to seasonally adjusted benchmark. \* = allowed error of one time period, ^ = average number of months until first detected. SA = seasonally adjusted, T = trend.

Figures 4a-4e show the proportion of false turning points for 59 simulated time series where turning points occurred in the most recent 6 months, between 7 and 12 months, and between 13-24 months. For example, the proportion of false turning points for turning points occurring in the most recent 6 months is described as the average proportion of turning points in the most recent 6 months in which turning points can be detected, eg. if  $k=m=3$  and December has just been observed, then we are considering turning points between April and September inclusive which do not match with benchmark turning points. Figures 4a, 4c and 4d show the distribution of the derived seasonally adjusted estimates against the proportion of false turning points.

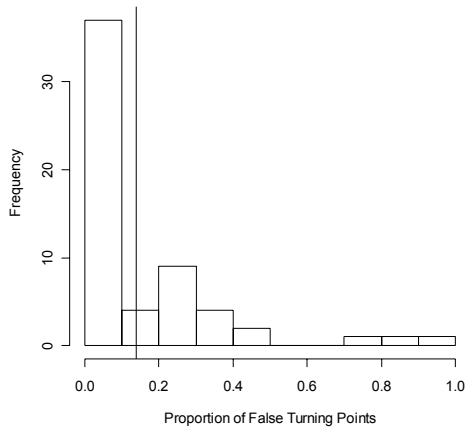


Figure 4a. Seasonally adjusted: Proportion of false turning points for turning points in the most recent 6 months for 59 simulated series.

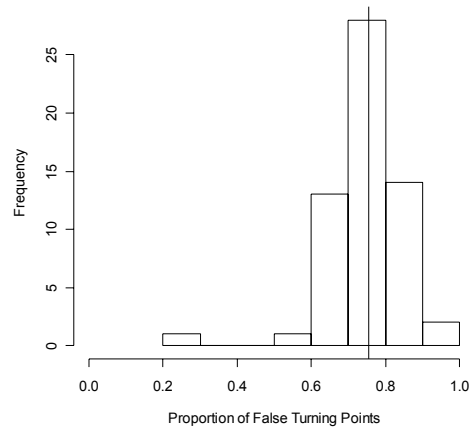


Figure 4b. Trend: Proportion of false turning points for turning points in the most recent 6 months for 59 simulated series.

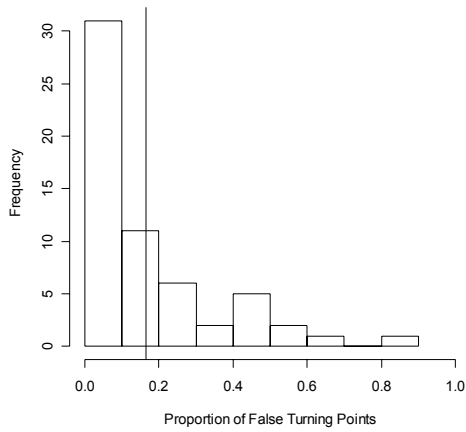


Figure 4c. Seasonally adjusted: Proportion of false turning points for turning points between 7 and 12 months for 59 simulated series.

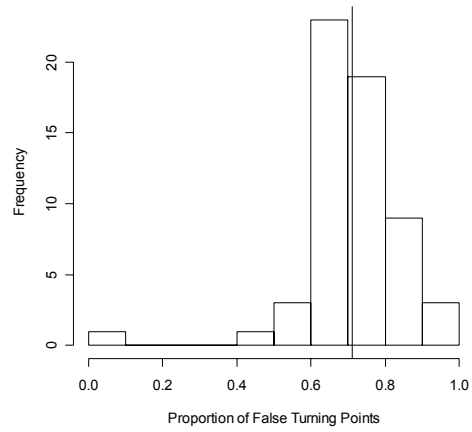


Figure 4d. Trend: Proportion of false turning points for turning points between 7 and 12 months for 59 simulated series.

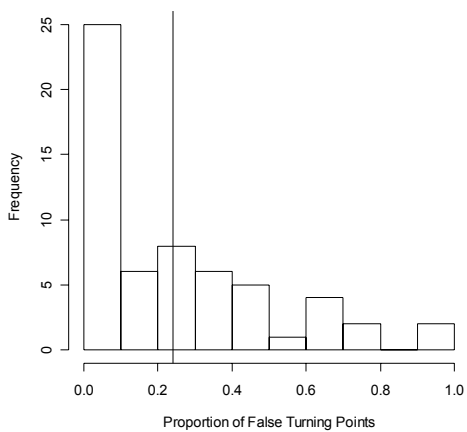


Figure 4e. Seasonally adjusted: Proportion of false turning points for turning points between 13 and 24 months for 59 simulated series.

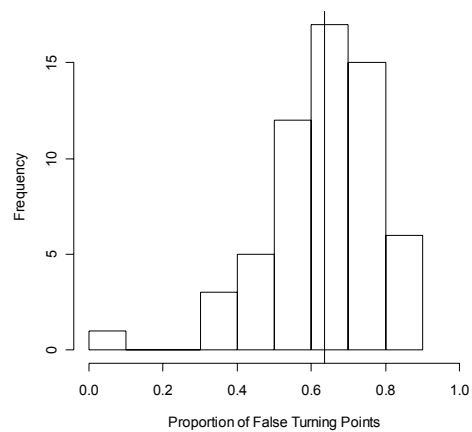


Figure 4f. Trend: Proportion of false turning points for turning points between 13 and 24 months for 59 simulated series.

The proportion of false turning points was observed to be much larger in the trend estimates than in seasonally adjusted estimates for the simulated time series. It is important to consider here that the number of turning points detected in seasonally adjusted estimates is small due to the nature of the turning point definition. This plays a role in influencing the probability of false turning points obtained. For example, if a time series has just one turning point detected which happens to be false, the proportion of false turning points for that series will be one. A well known property of trend estimates produced from the X-11 process is the “wiggly” trend (Dagum, 1996). This property can be caused by the nature of the trend filters used within the X-11 process (Henderson, 1916) and the influence of the sample design which can induce correlation between time series estimates.

Table 2 gives a comparison of the percentage of turning points detected and not detected in the seasonally adjusted and trend estimates when compared against the respective benchmark. For example, column 2 (% T|T) gives the percentage of points detected as true turning point in the detection series and benchmark series for the seasonally adjusted estimates. Columns 2 and 3 show that the trend estimates have a higher percentage of determining the true turning point. Columns 3 and 4 show that the seasonally adjusted estimates have a higher percentage of missing turning points when compared to the trend estimates. Columns 6 to 9 show that the trend estimates can give an increased percentage of detecting false turning points when compared to the seasonally adjusted estimates. Figure 5a and 5b shows boxplots of the percentage of time points detected as turning points in the detection series and benchmark series for the simulated time series.

Series	% T   T		% F   T		% F   F		% T   F	
	SA	Trend	SA	Trend	SA	Trend	SA	Trend
Employed persons	76.6	78.9	23.4	21.1	99.7	99.5	0.3	0.5
Unemployed persons	38.9	85.0	61.1	15.1	100.0	98.8	0.0	1.2
Short term visitor arrivals	82.8	84.8	17.2	15.3	99.8	98.2	0.2	1.8
Simulated (mean)	38.6	74.9	61.4	25.2	99.6	95.9	0.4	4.1

Table 2. The percentage of time points detected as turning points in detection series and benchmark series (detection | benchmark) where F = False detection and T = true detection for seasonally adjusted (SA) and trend estimates.

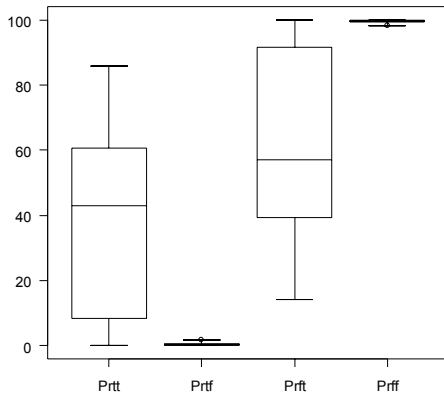


Figure 5a. Seasonally adjusted: Percentage of time points detected as turning points for combinations of false and true detection when compared against benchmark.

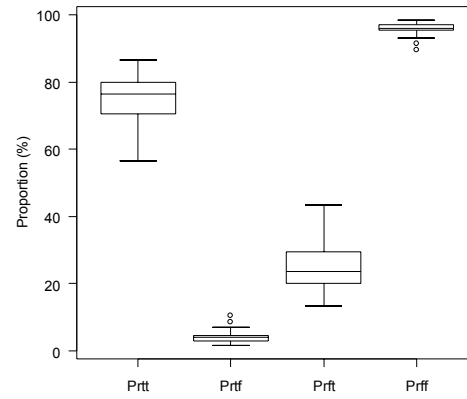


Figure 5b. Trend: Percentage of time points detected as turning points for combinations of false and true detection when compared against benchmark.

## 6 Comments

Users of official statistics can choose to use the original, seasonally adjusted or trend estimates either individually or in combination to aid in the decision making process. The seasonally adjusted estimates are reasonably well defined and the concept of seasonal adjustment is understood. Trend estimates are not well defined and different trend estimates can be derived from the same time series depending on the time frame of interest and understanding of a trend. Trend estimates produced as part of a standard seasonal adjustment process provide a standard definition for a trend and may be more appropriate to aid in assessment of the underlying direction of the original time series. Trend estimates are needed, and used, as part of the X-11 process. This study considered seasonally adjusted and trend estimates produced from an X-11 process.

Users need to be aware of the availability and limitations of all of the available time series estimates and make an appropriate and informed choice on which estimate to use. This initial study shows that there is a trade-off between timeliness of detection and false turning points for different time series estimates. If timeliness is important then the trend estimates provide the most appropriate measure to determine turning points in a timely fashion with the possibility of an increase in the number of false turning points. If minimising the detection of false turning points is important then seasonally adjusted estimates are appropriate with the drawback that turning points may take longer to be determined. The nature of the volatility of the time series is important to consider in assessing the reliability of the detection of turning points.

The turning point definition used in this paper is widely used in the literature. However, it is not robust to handle a volatile seasonally adjusted time series. Therefore, the results presented in this paper may be biased in favour of the trend estimates due to the strict monotonic definition. A more robust definition for a turning point is under study in order to take the volatility of a time series into account. See ABS (2006c) for further results which use the alternative Phase-Average-Trend turning point definition.

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