

# Benchmarking of Israeli economic time series and seasonal adjustment



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**Benchmarking of Israeli Economic Time Series  
and Seasonal Adjustment**

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## Abstract

Benchmarking deals with the problem of combining a series of high-frequency data (e.g., monthly) with a series of low frequency data (e.g., quarterly) into a consistent time series. When discrepancies arise between the two series the latter is usually assumed to provide more reliable information.

There are two main approaches to benchmarking of time series: a purely numerical approach and a statistical modeling approach. The numerical approach encompasses the family of least-square minimization methods (Denton, 1971). This benchmarking procedure is based on a movement preservation principle that is widely used by government statistical agencies and central banks around the world.

Statistical modeling approaches include ARIMA-based methods proposed by Hillmer and Trabelsi (1987, 1990), regression models (Cholette and Dagum, 1991), state space models (Durbin and Quenneville, 1997).

At the Central Bureau of Statistics (CBS) Israel, the need for the application of benchmarking techniques arises, for example for Labour Force Survey data and for National Accounts estimates, where monthly or quarterly and quarterly or annual data, respectively, may show inconsistent movements. Generally, the binding benchmarking technique seems to be the most appropriate for a statistical agency for solving the problem of discrepancies in the original administrative data, and the unbinding methods may be useful for survey data with known standard deviation. However, since these series are seasonally adjusted anew each month (quarter) several problems may arise because of the filtering process. Furthermore, when a system of time series is seasonally adjusted, the accounting constraints that link the original series may no longer be fulfilled.

In this paper, an empirical study that compares between the Denton approach and the regression approach is carried out for several Labour Force series, when a special emphasis is put on issues concerning seasonal adjustment: seasonal pattern preservation and the quality of the seasonal adjustment. In addition, the seasonal adjustment and the benchmarking of the composite series are considered when dealing with a system of time series. The indirect benchmarking method is proposed for benchmarking the composite and the component series. The statistical quality issues related to the benchmarking procedure and seasonal adjustment are discussed.

*Keywords:* Benchmarking methods, Denton's movement preservation method, Regression approach, Seasonal adjustment, Statistical data quality.

## 1. Introduction

Benchmarking is an important problem faced by the statistical agencies. For a target socioeconomic variable, two sources of data, for example, an annual administrative data and a quarterly repeated survey data, may be available. When discrepancies between the series of high frequency (e.g. quarterly) data and a series of low frequency (e.g. annual) data arise the latter is usually assumed to provide more reliable information. The problem of adjusting the monthly or quarterly time series to make them consistent with the quarterly or annual totals is known as benchmarking.

There are two main approaches to benchmarking of time series: a purely numerical approach and a statistical approach. The first includes the family of least-square minimization methods and is based on a movement preservation principle (Denton, 1971). The latter includes ARIMA-based signal extraction method (Hillmer and Trabelsi, 1987, 1990), regression models (Cholette and Dagum, 1994) and state-space models (Durbin and Quenneville, 1997).

Formally, let  $Y_t$  be the series of high frequency:

$$Y_t = \theta_t + e_t, \quad t = 1, \dots, T, \quad (1)$$

and  $Z_m$  be the stock series of low frequency:

$$Z_m = \frac{1}{p_m} \sum_{t \in m} \theta_t + w_m, \quad m = 1, \dots, M, \quad (2)$$

where  $e_t$  and  $w_m$  are the respective survey errors and  $t \in m$  means that  $t$  is included in the time period covered by  $Z_m$ , and  $p_m$  is the number of high-frequency observations covered by  $Z_m$ . For stock series  $p_m \equiv 1, \forall m$ .

Forcing monthly or quarterly figures to be equal to the given quarterly or annual totals (benchmark), leads to the binding benchmarking technique that seems to be the most appropriate for certain series, e.g. the national accounts series, published by the statistical agency. By setting  $w_m = 0, \forall m$ , in (2) the binding benchmarking is realized and the high-frequency time series is revised under the assumption that the low-frequency data is "true" in terms of survey errors. Otherwise, the benchmarking is unbinding. In this case one can adjust the original high-frequency data using the external data (e.g. quarterly or annual totals), under some accounting constraints, but without forcing the sums to match exactly the low-frequency data of the relevant time

period. The main advantage of this method is that it exploits the additional information about the autocorrelation structure of the benchmarks. As a result, the estimation errors from the unbinding benchmarking techniques usually are smaller. The unbinding technique may be applied to survey data, for which the assumption about zero variance of benchmark series is too strong.

At the Central Bureau of Statistics (CBS), Israel, the need for the application of benchmarking techniques arises, mainly, for Labour Force Survey Data and for National Accounts estimates, where monthly or quarterly and quarterly or annual data, respectively, may show inconsistent movements. However, several problems in seasonal adjustment and in benchmarking of the systems of time series may arise because of the filtering process.

At the CBS, seasonal adjustment is carried out using the X-12-ARIMA program that was developed by the US Census, 1998. This method is based on moving averages filtering process for the estimation of seasonal and trend-cycle components of time series. Two main problems in benchmarking procedure arise using this method:

1. The concurrent seasonal adjustment process in use at the CBS introduces revisions in the seasonally adjusted series and the trend-cycle estimates, since these series are seasonally adjusted anew each month or quarter. Benchmarking procedure also causes revisions in the original data, and together with the seasonal adjustment procedure they may increase the size of these revisions. As a result, the seasonal pattern of the series might be affected due to these revisions.
2. When a system of time series is seasonally adjusted, the accounting constraints that link the original series may no longer be fulfilled. Applying benchmarking process, which also affects these links, may increase the size of the discrepancies.

In this paper, the Labour Force Survey data is analyzed. The Israel Labour Force Survey is carried out by the CBS and serves as a source of data on the labour force characteristics and trends as well as household economics and demographics. The survey is designed as an annual survey of a nationally representative sample of approximately 12,000 households, and about 23,000 different households are interviewed every year. The survey population includes the whole permanent population of Israel aged 15 and over. The rotation panel structure of the survey provides repeated measurements on the same sample at different time points during

18 months and leads to 50% overlap between two consecutive quarters. This enables estimation of gross changes between the quarters. The CBS also produces very limited monthly data on labour force characteristics; mainly, number of employed and unemployed persons, by sex.

The empirical study focuses on issues concerning with seasonal adjustment and a special emphasis is put on data quality issues. Revisions in the seasonally adjusted series before and after benchmarking and the seasonal patterns of the original and the benchmarked series for several benchmarking techniques are studied. The direct and indirect benchmarking methods of a system of time series are compared, similarly to direct and indirect seasonal adjustment methods that are well known. Several accounting links preservation methods are examined, and a solution for the method based on the regression approach is provided.

## 2. Benchmarking methods

As stated above, several approaches exist to the benchmarking problem. In this section two main benchmarking methods that are relevant to the official statistics are reviewed. Let us point out that other benchmarking methods were developed, such as state space model (Pfefferman and Burck, 1990, Durbin and Quenneville, 1997) and models based on small area estimation (Pfefferman , 2002, Pfefferman and Tiller, 2003).

### 2.1. Denton's benchmarking method

The Denton family of least-squares based benchmarking methods is widely used by statistical agencies and international institutes around the world to solve the benchmarking problem (IMF, 2002). The general objective of this technique is a) maximum preservation of the short-term movements in the original (indicator) series and b) to obtain exact benchmark to total (binding). The estimation of a benchmarked series is based upon the minimization of a penalty function subject to a given set of benchmarking constrains. Let  $L(\theta, \lambda)$  be Lagrangian function:

$$L(\theta, \lambda) = (\theta - Y)' A(\theta - Y) + \lambda' (Z - L\theta) \quad (3)$$

where  $Y$  denotes the original series,  $\theta$  defines the benchmarked series to be estimated,

$Z$  is the low-frequency benchmarks,  $L$  is a design matrix that defines the links between the original series and the benchmarks, and  $A$  is a matrix used to define the special objective function. The solution for (3) is provided by:

$$\hat{\theta} = Y + A^{-1}L'(LA^{-1}L')^{-1}(Z - LY) \quad (4)$$

The two main variants of Denton's benchmarking procedure are:

1. The additive method that preserves the simple period-to-period change:

$$(Y_t - Y_{t-1}) - (\theta_t - \theta_{t-1}) \equiv (Y_t - \theta_t) - (Y_{t-1} - \theta_{t-1}), \quad t \in m, \quad m = 1, \dots, M, \quad t = 1, \dots, T$$

The objective function to be minimized for the first differences is thus given by:

$$\min_{\theta} \sum_{t=2}^T [(Y_t - \theta_t) - (Y_{t-1} - \theta_{t-1})]^2 \quad (5)$$

2. The proportional method that preserves the proportional period-to-period change:

$$\frac{Y_t - \theta_t}{\theta_t} - \frac{Y_{t-1} - \theta_{t-1}}{\theta_{t-1}} \equiv \frac{Y_t}{\theta_t} - \frac{Y_{t-1}}{\theta_{t-1}}, \quad t \in m, \quad m = 1, \dots, M, \quad t = 1, \dots, T$$

The objective function to be minimized for the first differences is thus given by:

$$\min_{\theta} \sum_{t=2}^T \left[ \left( \frac{Y_t}{\theta_t} \right) - \left( \frac{Y_{t-1}}{\theta_{t-1}} \right) \right]^2 \quad (6)$$

One can think about other forms of the objective functions, as it has been shown by Denton (1971), Helfand, Monsour and Trager (1977), Fernandez (1981) and Cholette (1979, 1984). Generally, the Denton technique is relatively simple, robust and well suited for large-scale applications. Moreover, the proportional method provides an effective framework for benchmarking, interpolation and extrapolation of time series that preserves month-to-month (quarter-to-quarter) changes in the data. On the other hand, these methods do not use any additional information in the data, such as correlation structure of the series or the survey errors.

## 2.2. Regression approach

The stochastic approach to benchmarking problem proposed by Hillmer and Trabelsi (1987, 1990) is well known as signal extraction method and it assumes that a signal  $\theta$  follows an ARIMA model. A more general regression approach was developed by



Cholette and Dagum (1994) and is implemented the BENCH program written by the Canadian staff (1994). The model consists of two linear equations:

$$\begin{aligned} Y_t &= a + \theta_t + e_t, \quad E(e_t) = 0, \quad E(e_t e_{t-k}) = \sigma_{e_t} \sigma_{e_{t-k}} \rho_k, \quad t = 1, \dots, T \\ Z_m &= \left( \sum_{t \in m} \theta_t \right) / p_m + w_m, \quad E(w_m) = 0, \quad E(w_m^2) = \sigma_{w_m}^2, \quad m = 1, \dots, M \end{aligned} \quad (7)$$

The equations (7) define the additive benchmarking model, where  $a$  is constant bias (intercept), and  $e_t$  are the autocorrelated errors that may be interpreted as survey errors. These errors may be heteroscedastic, i.e. the variance  $\sigma_{e_t}^2$  may vary with time  $t$ . The autocorrelations  $\rho_k$  correspond to those of a stationary and invertible Auto Regressive Moving Average (ARMA) process, supplied by the user. This is equivalent to assuming that  $e_t$  follows a process given by:

$$e_t = \sigma_t \varepsilon_t,$$

where  $\varepsilon_t$  follows the selected stationary ARMA process:

$$\phi_p(B)\varepsilon_t = \eta_q(B)v_t,$$

where  $B$  is the backshift operator,  $\phi_p(B)$  and  $\eta_q(B)$  are the autoregressive (AR) and moving average (MA) polynomials respectively and  $v_t$  is white noise.

Replacing the first equation in (7) by:

$$\ln Y_t = \ln(a_t \times \theta_t \times e_t) = \ln a + \ln \theta_t + \ln e_t \quad (7a)$$

leads to the multiplicative (log-additive) model, and if this model is replaced by:

$$Y_t = a \times \theta_t + e_t \quad (7b)$$

the mixed benchmarking model is obtained. The GLS solution for all these models are given in Cholette and Dagum (1994). The binding regression method is achieved by setting  $w_m = 0, \forall m$ . It may be shown that either additive or multiplicative Denton technique is a special case of the regression method under the following assumptions:

1.  $e_t$  follows the random walk process;
2.  $w_m = 0, \forall m$ , i.e. the benchmarking is binding;
3. variances of high-frequency data  $Y_t$  are constant;
4. bias parameter  $a$  is omitted.

Under these assumptions the additive regression model corresponds to the additive Denton method and the multiplicative regression model corresponds to the proportional Denton method.

Cholette and Dagum (1994) showed that the regression framework provides BLUE estimates. Also, the method has several desirable features: it allows to adjust all types of time series, it takes into account a stochastic structure of the series, it preserves movements of the original series, minimizes revisions in the data, and some other. When the binding possibility may be preferable for the national account series with an administrative indicator series of benchmarks, the unbinding regression method may be useful for survey data for which the assumption of zero variance of the low-frequency data may be unreasonable.

### **3. Issues on Seasonal Adjustment**

#### **3.1. Seasonal Adjustment in CBS**

More than 400 monthly and quarterly time series are seasonally adjusted at the CBS. The time series collected by the CBS are statistical records of a particular social or economic activity, like industrial production, person-nights in tourist hotels, labour force characteristics. They are measured at regular intervals of time, usually monthly or quarterly, over relatively long periods. This allows to disclose patterns of behavior over time, to analyze them and place the current estimates into a more meaningful and historical perspective. All the series are seasonally adjusted by the X-12-ARIMA program, which became the standard program for seasonal adjustment in 2004.

A time series can be decomposed into a number of fundamental components, each of which has its own distinguishing character. In a simple model, the original data at any time point (denoted by  $O_t$ ) may be expressed as a function  $f$  of three main components: the seasonality ( $S_t$ ), the trend-cycle ( $C_t$ ), and the irregularity ( $I_t$ ), that is:

$$O_t = f(S_t, C_t, I_t). \quad (8)$$

Depending mainly on the nature of the seasonal movements of a given series, several different models can be used to describe the way in which the components  $C_t$ ,  $S_t$ , and  $I_t$  are combined to compose the original series  $O_t$ . The multiplicative model:

$$O_t = S_t \times C_t \times I_t$$

treats all three components as dependent of each other; that is, the seasonal oscillation size increases and decreases with the level of the series. The irregular factors  $I_t$  may be decomposed into sub-components: the changes in the number of trading days and festival dates (also called calendar effects), and the remaining irregularity. Therefore the model may be extended as follows:

$$I_t = P_t \times E_t$$

and thus:

$$O_t = C_t \times S_t \times P_t \times E_t$$

where,  $P_t$  is the adjustment factor for the calendar effects (changes in the festival dates and the number of trading days in a month) and  $E_t$  is the residual variation caused by all other influences. Most series at the CBS, including National Accounts and Labour Force Survey series, are adjusted multiplicatively.

### **3.2. Concurrent Seasonal Adjustment**

Concurrent Seasonal Adjustment means that the seasonal adjusted series is calculated anew each month (quarter) on the basis of data that includes the current (new) observation. In general, new data contribute new information about changes in the seasonal pattern that preferably should be incorporated as early as possible. When concurrent seasonal adjustment is applied to a series, the seasonally adjusted values for the whole series, including the most recent month (quarter), are obtained directly from the seasonal adjustment procedure without the use of forecast seasonal and prior adjustment factors. Empirical studies have shown that the revisions to the seasonally adjusted data are smaller for series seasonally adjusted using the concurrent method. Concurrent seasonal adjustment is applied to all series published in the Monthly Bulletin of Statistics and in other publications of the CBS.

### **3.3. Seasonal Adjustment of Composite Time Series**

An aggregate (composite) series is a series that is composed of several sub-series (components). The direct seasonal adjustment method means seasonally adjusting the composite and the component series independently. Whereas, the indirect seasonal adjustment method consists of seasonally adjusting each component series and then obtaining the seasonally adjusted composite series as their sum.

The advantage of the indirect method is that the changes in the composite series can be broken down into the changes of the component series. Those component series for which the relative contribution of seasonality is low and/or the contribution of the irregular component is high, are included in the sum as unadjusted series. The monthly labour force series are adjusted using the indirect method.

## **4. Benchmarking and Seasonal Adjustment – Problems**

Benchmarking of the original time series may introduce several problems in seasonal adjustment process.

### **4.1. Single series**

- a) The seasonal factors, which are estimated from the decomposition model (8) by the filtering process, may change by pre-adjusting the series to benchmark values. Benchmarking procedure may cause a significant increase in the variance of the irregular component of the original time series if outliers exist in the benchmark series. In the case of benchmarking of the monthly series by quarterly benchmarks, revisions in seasonal pattern of the series may be caused by seasonal and irregular fluctuations in the quarterly data. These changes may affect the estimation of seasonal factors and, thus, seasonally adjusted series. All these changes are undesirable and in this case a benchmarking model that preserves the seasonal pattern of the original data is preferable to others.
- b) In general, since the benchmarking may affect the irregular component of a time series, the trading day and holiday effects ( $P_t$ ) estimation may also be affected. In this paper the impact of benchmarking on this estimation is not

investigated, but still it must be pointed out that in a series with significant trading day and holiday influences, large changes in the irregular component may seriously affect the estimation of  $P_t$ .

- c) The estimated seasonal factors and seasonally adjusted series are subject to revisions every month or quarter under concurrent seasonal adjustment method. Since this method is applied to all series published by the CBS, the benchmarking revisions become more critical. Large revisions in the original data each month (quarter) due to computational processes are a problem, and we would prefer a benchmarking method with relatively small size of revisions.

## **4.2. Composite series**

- a) Applying the seasonal adjustment procedure to a system of time series, where a number of sub-series are summed up to form the aggregate (composite) series, breaks off the links and the accounting constraints are no more fulfilled. On the other hand, applying the benchmarking procedure directly to the sub-series and to the grand-total series also violates these constraints. As was pointed out in section 3.3, the indirect seasonal adjustment method successfully avoids this danger without affecting, in many cases, the quality of the seasonally adjusted series. The same principle may be applied to the benchmarking: it is trivial that if all sub-series are benchmarked directly and the composite series obtained by summing them up will also be benchmarked. Consequently, one has to develop the criteria for selecting the indirect and the direct benchmarking methods for a specific system of time series.
- b) When the indirect procedure cannot be applied due to statistical or econometrical reasons to a system of time series, the account links preservation problem arises. Di Fonzo and Marini (2003) proposed a benchmarking method for adjusting a system of seasonally adjusted time series, using the Denton technique. If another benchmarking method seems to be appropriate, a more general approach must be developed.

### **4.2.1. Methods for preservation of accounting links**

Here we summarize several approaches that preserve the accounting links in benchmarking and propose a technique based on the regression principle. The indirect benchmarking method preserves the accounting links between sub-series and composite series. An empirical analysis that compares direct to the indirect benchmarking is presented in section 5.3. The indirect benchmarking method is consistent with indirect seasonal adjustment in terms of seasonal pattern preservation in the sub-series, where the seasonal fluctuations in the sub-series are not swallowed up in the composite data.

If, for some reason, the direct adjustment is applied to all series in the system, the accounting constraints that linked the original series will be violated after benchmarking procedure. Several methods are considered in order to overcome this type of problem. First, one can simply divide amongst the sub-series the difference between the sum of the benchmarked sub-series and benchmarked aggregate series, for example, by the weights of the sub-series. This intuitive approach may be useful in some cases, for example, in benchmarking of two different systems of time series when the aggregate data has two different breakdowns. An example of this in the CBS is the composite monthly series of employed persons, by sex and by economic branch. For monthly employed males and females and for their totals we have quarterly benchmarks, but we have no additional information about the latter set of series. The proposed algorithm that preserves the accounting links for these series is as follows:

- a) Apply the benchmarking procedure that preserves the accounting links to the first system (i.e. employed persons by sex) to obtain the benchmarked composite series (for example, by applying indirect method).
- b) Calculate the difference between the benchmarked composite series and the sum of the sub-series from the second system.
- c) Add to each sub-series the relative difference, based on the weight of the sub-series in the aggregate figure. Calculating weights for each observation rather than using the average weight value for all observations will cause smaller revisions and better preservation of month-to-month rates of change in the sub-series.

This very simple technique may also be useful in adjustment of single system of time series, when the benchmark values are available only for the composite data.

Obviously, we need a more sophisticated approach for the case when benchmarks are available for all series and direct benchmarking technique seems to be appropriate. Di

Fonzo and Marini (2003) presented accounting link preservation method based on Denton approach. We propose here a more general approach based on the regression principle to preserve the accounting links between the sub-series.

Let us consider the following system:

$$\begin{aligned}
Y_t &= a + \theta_t + e_t, \quad E(e_t) = 0, \quad E(e_t e_{t-k}) = \sigma_{e_t} \sigma_{e_{t-k}} \rho_k, \quad t = 1, \dots, T \\
Y_t^{(1)} &= \alpha^{(1)} + \theta_t^{(1)} + e_t^{(1)}, \quad E(e_t^{(1)}) = 0, \quad E(e_t^{(1)} e_{t-k}^{(1)}) = \sigma_{e_t^{(1)}} \sigma_{e_{t-k}^{(1)}} \rho_k^{(1)}, \quad t = 1, \dots, T \\
Y_t^{(2)} &= a^{(2)} + \theta_t^{(2)} + e_t^{(2)}, \quad E(e_t^{(2)}) = 0, \quad E(e_t^{(2)} e_{t-k}^{(2)}) = \sigma_{e_t^{(2)}} \sigma_{e_{t-k}^{(2)}} \rho_k^{(2)}, \quad t = 1, \dots, T \\
s.t. \\
Z_m &= \frac{1}{p_m} \sum_{t \in m} \theta_t + w_m, \quad E(w_m) = 0, \quad E(w_m)^2 = \sigma_{w_m}^2, \quad m = 1, \dots, M \\
Z_m^{(1)} &= \frac{1}{p_m} \sum_{t \in m} \theta_t^{(1)} + w_m^{(1)}, \quad E(w_m^{(1)}) = 0, \quad E(w_m^{(1)})^2 = \sigma_{w_m^{(1)}}^2, \quad m = 1, \dots, M \\
Z_m^{(2)} &= \frac{1}{p_m} \sum_{t \in m} \theta_t^{(2)} + w_m^{(2)}, \quad E(w_m^{(2)}) = 0, \quad E(w_m^{(2)})^2 = \sigma_{w_m^{(2)}}^2, \quad m = 1, \dots, M \\
\theta_t &= \theta_t^{(1)} + \theta_t^{(2)}
\end{aligned} \tag{9}$$

where  $Y_t, Y_t^{(1)}, Y_t^{(2)}$  are high-frequency composite and the two component time series, respectively,  $Z_m, Z_m^{(1)}, Z_m^{(2)}$  are low-frequency composite and the two sub-series at time  $m$ , respectively,  $\theta_t, \theta_t^{(1)}, \theta_t^{(2)}$  are the benchmarked series to be estimated, and  $e_t, e_t^{(1)}, e_t^{(2)}, w_m, w_m^{(1)}, w_m^{(2)}$  are the corresponding model errors, that may follow some ARMA process.  $\sigma_{e_t}, \sigma_{e_t^{(1)}}, \sigma_{e_t^{(2)}}$  may be considered as survey errors, and  $\rho_k, \rho_k^{(1)}, \rho_k^{(2)}$  as correlation parameters between the errors at times  $t$  and  $t-k$ . The value of  $p_m$  defines the period of high-frequency series that is covered by the benchmark  $Z_m$ . In the case of monthly series and quarterly benchmarks  $p_m = 3, \forall m$ . The last equation simply defines the accounting constraint for the composite time series. Note that this equation may be more complex, for example in the case of weighted composite series. Note also that the model for the errors of the composite series is derived from the models for the component series. Let us mention that the system (9) can be applied to any system with  $n$  component series.

Define:

1.  $I_{T \times T}$  - identity matrix;

2.  $J_{M \times T}$  - matrix in which:  $J_{m \times t} = \begin{cases} 1/p_m, t \in m \\ 0, t \notin m \end{cases}$

3.  $u_t$  - vector of ones.

We can rewrite the model in (9) in the matrix notation as follows:

$$Y^* = X\beta + v \quad (10)$$

where:

$$Y^* = \begin{bmatrix} Y \\ Y^{(1)} \\ Y^{(2)} \\ Z \\ Z^{(1)} \\ Z^{(2)} \end{bmatrix}_{(3T+3M) \times 1}, \quad v = \begin{bmatrix} e \\ e^{(1)} \\ e^{(2)} \\ w \\ w^{(1)} \\ w^{(2)} \end{bmatrix}_{(3T+3M) \times 1}, \quad \beta = \begin{bmatrix} a \\ a^{(1)} \\ a^{(2)} \\ \theta^{(1)} \\ \theta^{(2)} \end{bmatrix}_{(2T+3) \times 1}$$

with the design matrix  $X$ :

$$X = \begin{bmatrix} u & 0 & 0 & I & I \\ 0 & u & 0 & I & 0 \\ 0 & 0 & u & 0 & I \\ 0 & 0 & 0 & J & J \\ 0 & 0 & 0 & J & 0 \\ 0 & 0 & 0 & 0 & J \end{bmatrix}_{(3T+3M) \times (2T+3)}$$

The correlation structure is defined by:

$$E(v) = 0, E(vv') = V = \begin{bmatrix} V_e & 0 \\ 0 & V_w \end{bmatrix}_{(3T+3M) \times (3T+3M)}$$

where  $V_e$  and  $V_w$  are the covariance matrices for the high-frequency and the low-frequency series, respectively. Note that  $X$  and  $V$  matrix are of full rank. The General Least Square estimates of the model parameters (10) are given by:

$$\hat{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}Y^*$$

$$Cov(\hat{\beta}) = (X'V^{-1}X)^{-1}$$

Simplification of this expression is possible in order to decrease the dimension, of this problem, as shown by Cholette and Dagum (1994). It was also shown that the above estimates are BLUE. The estimation of the multiplicative and mixed regression models can be carried out by applying the same principles as in the additive case after some transformation (log-transformation for the multiplicative case), as shown by the



Canadian Staff (1994). Denton benchmarking procedure is equivalent when the following settings are applied:

1.  $e_t, e_t^{(1)}, e_t^{(2)}$  follow the random walk model,
2. intercepts  $a, a^{(1)}, a^{(2)}$  are omitted,
3. benchmarking is binding, i.e.  $w_m, w_m^{(1)}, w_m^{(2)} \equiv 0$
4. appropriate regression model is chosen (additive for Denton additive method and multiplicative for Denton proportional method).

## 5. Empirical study

The empirical study presented here includes a comparison between several benchmarking models for the labour force series and it focuses on seasonal adjustment. The requirement from the benchmarking model is such that it does not cause, as much as possible, distortion of the seasonal pattern of the series, and does not significantly affect the irregular movements. Note that all benchmarking calculations were carried out by the BENCH program (Canadian Staff, 1994), and the seasonal adjustment figures were obtained using the X-12-ARIMA program (US Census, 1998).

### 5.1. Benchmarking models and Seasonal Adjustment

In the following, we examine five binding and three unbinding benchmarking methods:

- 1) Denton additive model (5).
- 2) Denton proportional model (6).
- 3) Binding, i.e.  $w_m = 0, \forall m$ , regression additive model (7), with ARMA model for  $e_t$ .
- 4) Binding, i.e.  $w_m = 0, \forall m$ , regression multiplicative model (7a), with ARMA model for  $e_t$ .
- 5) Binding, i.e.  $w_m = 0, \forall m$ , regression mixed model (7b), with ARMA model for  $e_t$ .
- 6) Unbinding regression additive model (7), with ARMA model for  $e_t$ .

- 7) Unbinding regression multiplicative model (7a), with ARMA model for  $e_t$ .
- 8) Unbinding regression mixed model (7b), with ARMA model for  $e_t$ .

For each model, the benchmarking procedure is applied to the original data, and then the seasonal adjustment program is executed. Note that the ARMA model is estimated from the irregular part of the time series. Four monthly labour force sub-series are analyzed: employed males, employed females, unemployed males, and unemployed females. Direct and indirect benchmarking and seasonal adjustment methods are compared for the composite series, Total unemployed persons.

## 5.2. Statistics for model comparison

### 5.2.1. Benchmarking statistics

The ARMA model for the errors  $e_t$  and the parameter values are estimated, for all relevant time series. Three main statistics are calculated for the benchmarked series:

- 1) Average standard deviation =  $\frac{1}{T} \sum_{t=1}^T std(\hat{\theta}_t)$
- 2) Movement preservation statistic, in percentages, defined by:

$$M = \frac{1}{T-1} \sum_{t=2}^T abs\left[\frac{Y_t}{Y_{t-1}} - \frac{\hat{\theta}_t}{\hat{\theta}_{t-1}}\right] * 100.$$

This statistic is calculated as the average of differences, in absolute values, between the month-to-month rate of change in the original and the benchmarked series. The value of  $M$  will be smaller for the model that preserves better the month-to-month rate of change.

- 3) Range of seasonality preservation. The difference between the highest (peak) and the lowest (trough) of the annual seasonal factors, expressed in percentages, is called the range of seasonality, and is a measure of the magnitude of the seasonal variation. Denote seasonal factors by  $S_t$ :

$$R = \text{Range of seasonality} = \max_{t=1, \dots, 12} (S_t) - \min_{t=1, \dots, 12} (S_t).$$

Using the range of seasonality, the Israeli series were classified into three groups: a) high seasonality (range greater than 50 %), b) intermediate (20-50 %), and c) low (less than 20 %). Note, that unemployed females series is of intermediate seasonality, and all other series are of low seasonality.

The important property of the benchmarking method is to preserve the range of seasonality of time series. Thus, we introduce the following statistic for preservation of range of seasonality:

$$DR = R \text{ of original series} - R \text{ of benchmarked time series.}$$

Thus, a smaller value of DR means a similar range of seasonality in the benchmarked series.

### 5.2.2. Seasonal adjustment statistics

The X-12-ARIMA program produces a set of standard statistics for fitting the ARIMA model and for the quality of the seasonal adjustment; all these statistics are described in the X-12-ARIMA manual and in Findley (1998). In this paper, we refer to several statistics that are widely used at the CBS when we apply the seasonal adjustment procedure to a specific series.

- 1) ARIMA model statistics: model coefficients and their significance, average absolute percentage error in within sample forecasts for last three years, Chi-Square probability for goodness of fit, normality of the residuals, and log-likelihood based statistics.
- 2) Seasonal adjustment statistics: F test for stable seasonality (table D8A), standard deviation of estimated seasonal factors (table D10) and irregular factors (table D13), relative contributions in percentages of the components to the variance in the month-to-month changes in the original series (table F2B), number of months for cyclical dominance MCD (table F2E), and quality assessment summary statistic Q (table F3).

A new measurement was added: smoothness of the seasonally adjusted series, in percentages, defined by:

$$Smoothness = \frac{1}{T-1} \sum_{t=2}^T abs\left[\left(\frac{Y_t}{Y_{t-1}} - 1\right)\right] * 100$$

The smoothness index is computed as the average of the absolute percent month-to-month change in the series.

### 5.3. Empirical Results

Table 1 to Table 4 present the statistics for the benchmarking and for the quality of the seasonal adjustment for four series: employed males, unemployed males, employed females, unemployed females, respectively.

As shown in these tables, the lowest average standard deviation for the estimated benchmarking values is obtained for the binding regression methods. In the series: employed females and unemployed females the average standard deviations of benchmarked figures are smaller for the unbinding regression models in comparison to the Denton methods, but greater than for the binding regression methods. For two other series unbinding regression models provide the largest standard deviation of the estimated benchmarked series.

The Denton methods provide the best moving preservation statistic among the binding methods with slight superiority of the Denton proportional method. The benchmarking, ARIMA model and seasonal adjustment diagnostics indicate that this model preserves movements in the analyzed series better than all other binding benchmarking models.

Table 1. Employed Males - Benchmarking and Seasonal Adjustment Statistics									
	Benchmarking Model								
	Not Benchmarked	Binding additive	Binding multiplicative	Binding mixed	Denton additive	Denton proportional	Unbinding additive	Unbinding multiplicative	Unbinding mixed
<b>Benchmarking</b>									
ARMA model for errors	---	(5,0)(0,0) <sub>12</sub>			(1,0)(0,0) <sub>12</sub>		(5,0)(0,0) <sub>12</sub>		
Estimated ARMA parameters	---	AR1=-0.649, AR2=-0.541, AR5=-0.178			AR1=0.999		AR1=-0.649, AR2=-0.541, AR5=-0.178		
Average standard deviation	710.14	617.77	619.30	619.43	711.95	710.16	3218.06	3221.93	3221.93
Movement preservation statistic	---	0.9142	0.9131	0.9084	0.2041	0.2044	0.0050	0.0002	0.0002
<b>ARIMA Model in X-12</b>									
ARIMA model	(0 1 2)(0 1 1) <sub>12</sub>	(2 1 2)(0 1 1) <sub>12</sub>	(2 1 2)(0 1 1) <sub>12</sub>	(2 1 2)(0 1 1) <sub>12</sub>	(0 1 2)(0 1 1) <sub>12</sub>	(0 1 2)(0 1 1) <sub>12</sub>	(0 1 2)(0 1 1) <sub>12</sub>	(0 1 2)(0 1 1) <sub>12</sub>	(0 1 2)(0 1 1) <sub>12</sub>
Average absolute percentage error in within-sample forecasts: last 3 years	1.51	1.69	1.69	1.69	1.49	1.49	1.50	1.51	1.51
Chi-Sq. Probability	9.44	18.83	18.67	18.76	10.08	10.04	9.44	9.44	9.44
<b>Quality of Seasonal Adjustment</b>									
F-test for seasonality (table D8A)	3.26	2.26	2.27	2.27	3.23	3.26	3.26	3.26	3.26
Std. of seasonal factors (table D10)	0.89	0.80	0.80	0.80	0.82	0.85	0.89	0.89	0.89
Std. of irregular factors (table D13)	0.92	1.11	1.11	1.11	0.88	0.90	0.92	0.92	0.92
Relative contributions of:									
I - irregular (table E3)	30.96	58.80	58.55	58.39	40.69	39.83	30.96	30.96	30.96
C - trend-cycle (table D12)	3.11	3.77	3.77	3.79	3.69	3.38	3.11	3.11	3.11
S - seasonality (table D10)	65.93	37.43	37.69	37.82	55.61	56.79	65.93	65.93	65.93
Range of Seasonality	3.06	2.97	2.98	2.97	2.75	2.69	3.05	3.06	3.06
Smoothness of seasonally adjusted series	1.45	1.45	1.84	1.84	1.84	1.43	1.45	1.45	1.45
MCD	5	5	5	5	5	5	5	5	5
Q	1.23	1.47	1.46	1.46	1.39	1.36	1.23	1.23	1.23

Table 2. Employed Females - Benchmarking and Seasonal Adjustment Statistics									
	Benchmarking Model								
	Not Benchmarked	Binding additive	Binding multiplicative	Binding mixed	Denton additive	Denton proportional	Unbinding additive	Unbinding multiplicative	Unbinding mixed
<b>Benchmarking</b>									
ARMA model for errors	---	(2,0)(0,0) <sub>12</sub>			(1,0)(0,0) <sub>12</sub>		(2,0)(0,0) <sub>12</sub>		
Estimated ARMA parameters	---	AR1=-0.894, AR2=-0.574			AR1=0.999		AR1=-0.894, AR2=-0.574		
Average standard deviation	25703.63	21867.64	21884.86	21893.97	25769.05	25704.29	25287.95	25317.57	25318.00
Movement preservation statistic	---	1.2845	1.2791	1.2822	0.2226	0.2240	0.2111	0.2118	0.2118
<b>ARIMA Model in X-12</b>									
ARIMA model	(2 1 0)(0 1 1) <sub>12</sub>	---	---	---	(2 1 0)(0 1 1) <sub>12</sub>	(2 1 0)(0 1 1) <sub>12</sub>	---	---	---
Average absolute percentage error in within-sample forecasts: last 3 years	1.30	---	---	---	1.15	1.15	---	---	---
Chi-Sq. Probability	7.31	---	---	---	7.51	7.87	---	---	---
<b>Quality of Seasonal Adjustment</b>									
F-test for seasonality (table D8A)	1.17	0.91	0.91	0.92	1.21	1.20	1.12	1.12	1.12
Std. of seasonal factors (table D10)	0.80	0.83	0.82	0.83	0.80	0.81	0.80	0.80	0.80
Std. of irregular factors (table D13)	1.12	1.30	1.30	1.30	1.16	1.16	1.14	1.14	1.14
Relative contributions of:									
I - irregular (table E3)	47.93	72.49	72.61	72.59	47.93	47.32	49.72	49.72	49.72
C - trend-cycle (table D12)	2.65	1.83	1.83	1.83	2.96	2.92	2.56	2.56	2.56
S - seasonality (table D10)	49.42	25.68	25.56	25.58	49.11	49.76	47.72	47.72	47.72
Range of Seasonality	3.51	3.39	3.37	3.39	3.43	3.48	3.51	3.51	3.51
Smoothness of seasonally adjusted series	1.86	1.86	2.33	2.33	2.33	1.86	1.86	1.93	1.93
MCD	6	6	6	6	6	6	6	6	6
Q	1.59	1.88	1.88	1.88	1.55	1.55	1.61	1.61	1.61

Table 3. Unemployed Males - Benchmarking and Seasonal Adjustment Statistics									
Benchmarking Model									
	Not Benchmarked	Binding additive	Binding multiplicative	Binding mixed	Denton additive	Denton proportional	Unbinding additive	Unbinding multiplicative	Unbinding mixed
<b>Benchmarking</b>									
ARMA model for errors	---	(2,0)(0,1) <sub>12</sub>			(1,0)(0,0) <sub>12</sub>		(2,0)(0,1) <sub>12</sub>		
Estimated ARMA parameters	---	AR1=0.458, AR2=-0.335 MA1=0.730			AR1=0.999		AR1=0.458, AR2=-0.335 MA1=0.730		
Average standard deviation	710.14	632.94	636.02	636.03	711.95	710.16	1396.63	1391.40	1391.40
Movement preservation statistic	---	1.9426	1.9236	1.9455	0.7360	0.7145	0.0499	0.0053	0.0053
<b>ARIMA Model in X-12</b>									
ARIMA model	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>
Average absolute percentage error in within-sample forecasts: last 3 years	9.87	9.36	9.44	9.31	9.26	9.23	9.82	9.87	9.87
Chi-Sq. Probability	26.59	5.78	6.20	6.16	18.10	19.60	26.57	26.54	26.53
<b>Quality of Seasonal Adjustment</b>									
F-test for seasonality (table D8A)	3.70	3.29	3.29	3.27	3.50	3.55	3.70	3.70	3.70
Std. of seasonal factors (table D10)	4.83	4.45	4.42	4.47	4.61	4.64	4.81	4.83	4.83
Std. of irregular factors (table D13)	5.89	5.99	5.95	6.01	5.82	5.75	5.86	5.89	5.89
Relative contributions of:									
I - irregular (table E3)	41.91	55.21	57.98	55.04	44.00	43.20	41.94	41.93	41.93
C - trend-cycle (table D12)	4.55	4.40	4.12	4.36	5.42	5.39	4.56	4.55	4.55
S - seasonality (table D10)	53.53	40.39	37.91	40.60	50.59	51.41	53.50	53.52	53.52
Range of Seasonality	17.85	15.50	15.60	15.52	16.15	16.36	17.77	17.85	17.85
Smoothness of seasonally adjusted series	7.84	7.84	7.95	8.00	7.97	7.85	7.84	7.80	7.83
MCD	8	11	11	11	6	6	8	8	8
Q	1.18	1.23	1.28	1.23	1.13	1.13	1.18	1.18	1.18

Table 4. Unemployed Females - Benchmarking and Seasonal Adjustment Statistics									
Benchmarking Model									
	Not Benchmarked	Binding additive	Binding multiplicative	Binding mixed	Denton additive	Denton proportional	Unbinding additive	Unbinding multiplicative	Unbinding mixed
<b>Benchmarking</b>									
ARMA model for errors	---	(0,1)(0,1) <sub>12</sub>			(1,0)(0,0) <sub>12</sub>		(0,1)(0,1) <sub>12</sub>		
Estimated ARMA parameters	---	MA1=0.808, MA12=0.622			AR1=0.999		MA1=0.808, MA12=0.622		
Average standard deviation	10530.87	8806.49	8763.20	8895.71	10557.67	10531.15	10133.73	10217.27	10216.11
Movement preservation statistic	---	4.2127	7.6874	4.6058	1.0277	0.9418	0.8926	0.9027	0.9012
<b>ARIMA Model in X-12</b>									
ARIMA model	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 2)(0 1 1) <sub>12</sub>	(0 1 2)(0 1 1) <sub>12</sub>	(0 1 2)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>	(0 1 1)(0 1 1) <sub>12</sub>
Average absolute percentage error in within-sample forecasts: last 3 years	7.99	8.64	8.68	8.77	8.01	8.06	7.97	8.04	8.04
Chi-Sq. Probability	81.21	89.26	66.70	89.61	81.05	79.10	84.59	84.77	84.77
<b>Quality of Seasonal Adjustment</b>									
F-test for seasonality (table D8A)	12.66	9.57	6.79	9.39	13.29	13.17	12.31	12.30	12.30
Std. of seasonal factors (table D10)	10.48	10.35	9.18	10.47	10.36	10.39	10.40	10.50	10.51
Std. of irregular factors (table D13)	6.29	7.29	8.92	7.37	6.28	6.38	6.31	6.38	6.38
Relative contributions of:									
I - irregular (table E3)	47.99	48.12	61.63	44.57	44.74	41.56	46.98	47.07	47.01
C - trend-cycle (table D12)	1.48	0.91	1.32	0.91	1.79	2.01	1.47	1.46	1.46
S - seasonality (table D10)	50.53	50.97	37.05	54.52	53.47	56.44	51.55	51.47	51.52
Range of Seasonality	31.69	32.09	27.69	32.45	29.18	29.20	31.68	32.00	32.02
Smoothness of seasonally adjusted series	9.69	9.69	11.52	14.49	11.89	9.46	9.69	9.83	9.94
MCD	9	12	12	12	8	8	9	9	9
Q	1.02	1.32	1.57	1.27	0.93	0.90	1.01	1.01	1.01

The best movement preservation statistic for benchmarked series and the best seasonal pattern preservation diagnostics among all analyzed methods is obtained for the unbinding methods. Applying the binding regression models may affect the ARIMA model estimation in the seasonal adjustment procedure, as it is observed in the employed females and unemployed females data.

For series with low range of seasonality (employed males, employed females and unemployed males) all the binding regression methods fail to preserve the movements in the original data and the seasonal pattern, whereas for the series with intermediate range of seasonality the additive and the mixed models provide better diagnostics for moving preservation and for quality of the seasonal adjustment.

### 5.3.1. Binding regression models

Diagram 1: Seasonal pattern for unemployed females  
Comparison between the binding regression benchmarking methods with ARMA option

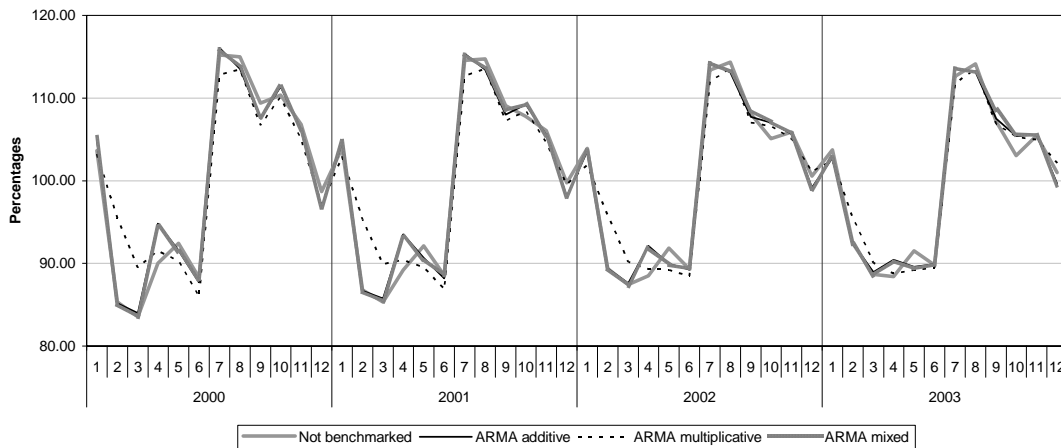


Diagram 1 illustrates the seasonal factors for the not benchmarked and the benchmarked series from the binding regression methods, for unemployed females, 2000-2003.

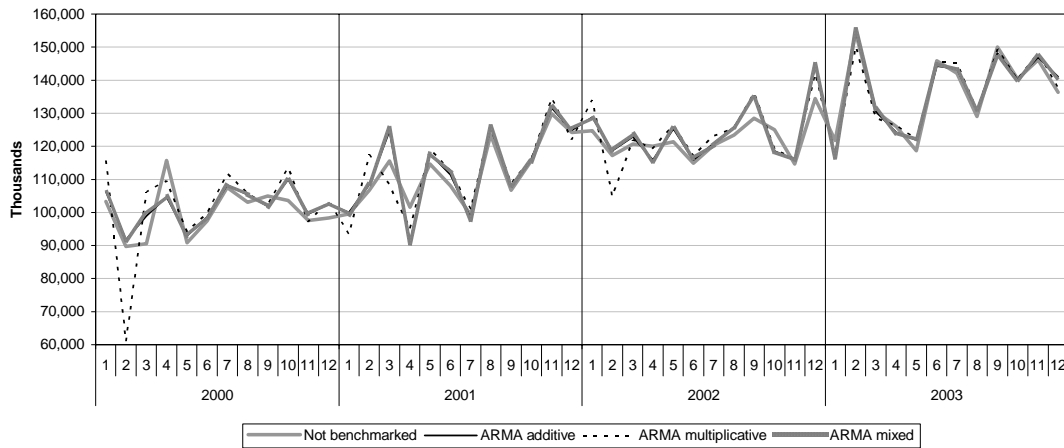
The binding regression benchmarking model applied to unemployed females series with intermediate range of seasonality, the seasonal pattern is significantly affected. This fact may as well be observed by the increase in the relative contribution of irregular component and decrease in the relative contribution of seasonal component of the series to the month-to-month changes in the original data (for example, from 48% to 73% in the employed females series, see Table 2). Consequently, as it shown in Table 2, Table 3, and Table 4, the value of F-test for presence of seasonality is lower, the standard deviation of irregular factors is higher, and the summary statistic Q is significantly higher than for all other benchmarking methods.

Diagram 2 illustrates the seasonally adjusted series for the not benchmarked and the benchmarked data from the binding regression methods, for unemployed females, 2000-2003.

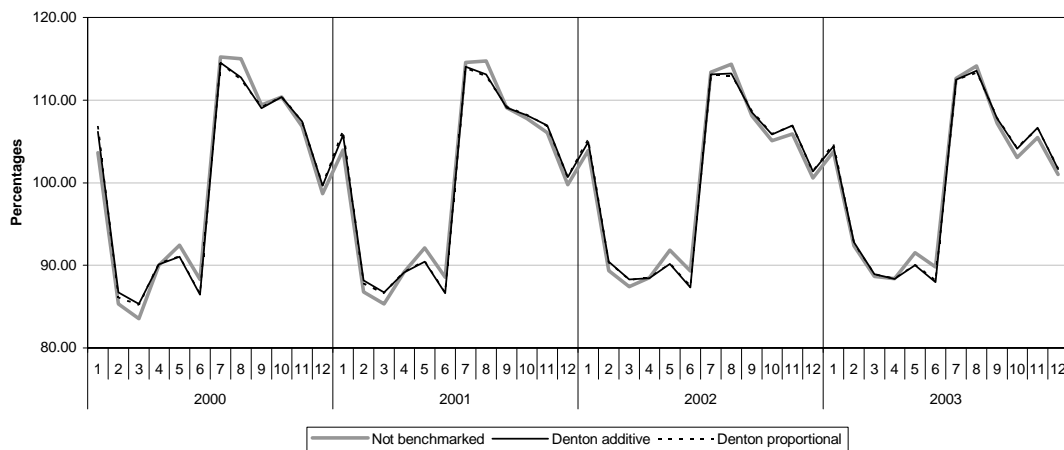
For all regression binding benchmarking methods, the unbenchmarked seasonally adjusted series is smoother than the benchmarked seasonally adjusted series. Only the multiplicative benchmarking model failed to preserve the seasonal pattern, the range of seasonality and the movements in the data for unemployed females, as shown in the Table 4. Additive and mixed benchmarking models preserve the seasonal pattern of this series and provide benchmarking and seasonal adjustment diagnostics that are

close to those of the original series. There are no significant differences between these two models.

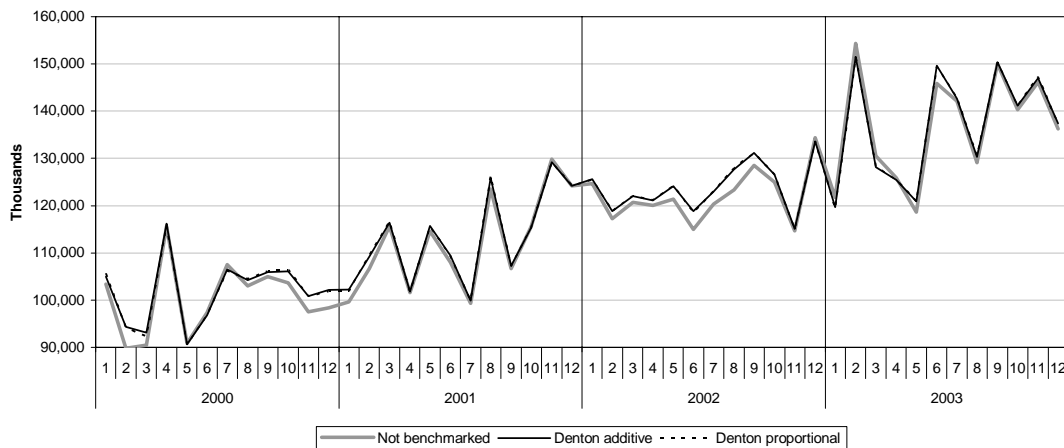
**Diagram 2: Seasonally adjusted series for unemployed females**  
**Comparison between the binding regression benchmarking methods with ARMA option**



**Diagram 3: Seasonal pattern for unemployed females**  
**Comparison between the Denton benchmarking methods**



**Diagram 4: Seasonally adjusted series for unemployed females**  
**Comparison between the Denton benchmarking methods**



### 5.3.2. Denton models

Diagram 3 illustrates the seasonal factors for the not benchmarked and the benchmarked series from the Denton methods, for unemployed females, 2000-2003.

Diagram 4 illustrates the seasonally adjusted series for the not benchmarked and the benchmarked data from the Denton methods, for unemployed females, 2000-2003.

In Diagram 3 and Diagram 4 show that the differences between the two Denton methods are not significant, especially for the seasonally adjusted series. As shown in Table 4, the proportional Denton method smoothed the unemployed females series and decreased the relative contribution of the irregular component, in comparison to all other benchmarking methods.

### 5.3.3. Unbinding regression models

Diagram 5 illustrates the seasonal factors for the not benchmarked and the benchmarked series from the unbinding regression methods, for unemployed females, 2000-2003.

Diagram 6 illustrates the seasonally adjusted series for the not benchmarked and the benchmarked data from the unbinding regression methods, for unemployed females, 2000-2003.

All three unbinding regression models provide the best movement preservation and seasonal adjustment diagnostics in the sense that the differences from the original data are smaller and the seasonal pattern is unchanged, as displayed in Table 4 and Diagram 5.

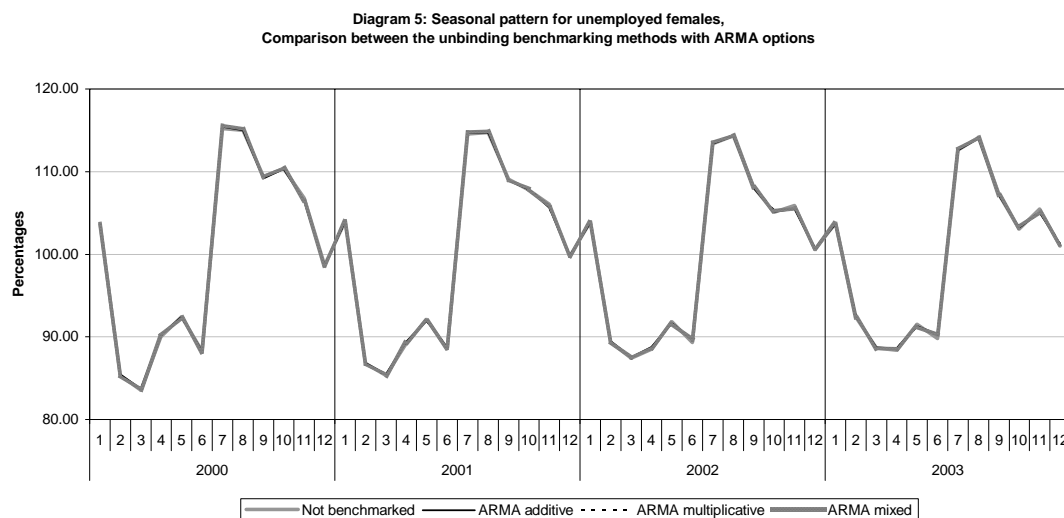




Diagram 6: Seasonally adjusted series for unemployed females.  
Comparison between the unbinding benchmarking methods with ARMA options

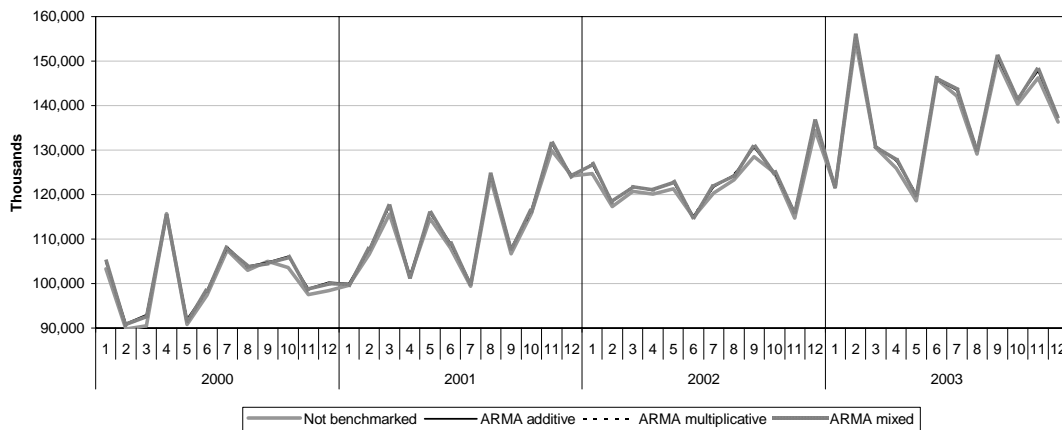


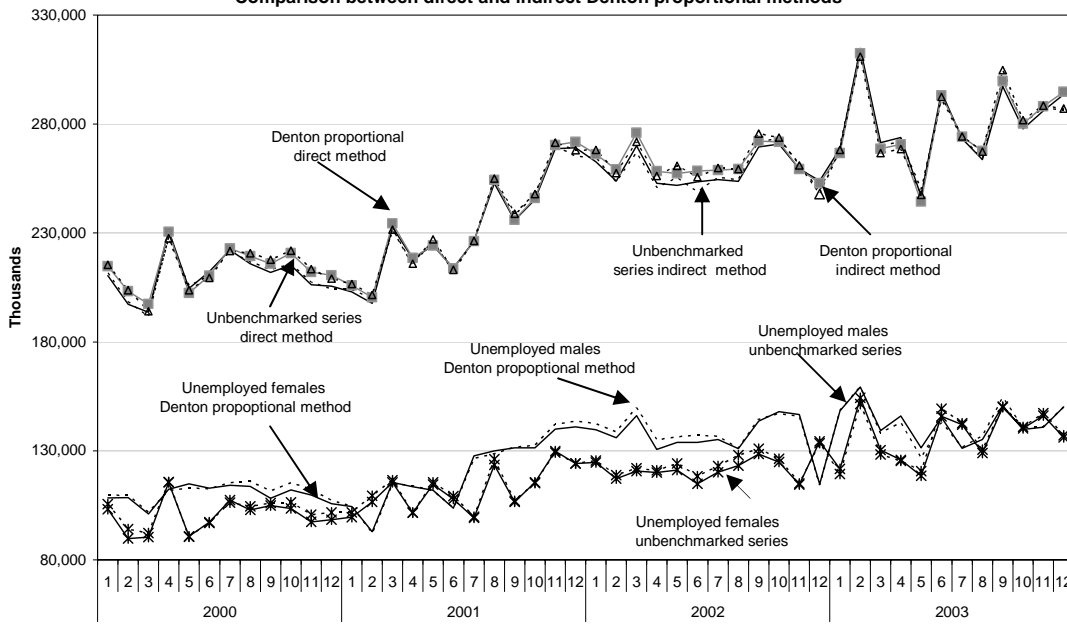
Diagram 6 shows that there are no significant differences between the seasonally adjusted series for unemployed females provided by the three unbinding regression models, and these series are very close to the not benchmarking seasonally adjusted data.

#### 5.3.4. Benchmarking of aggregate series – Total Unemployed Persons

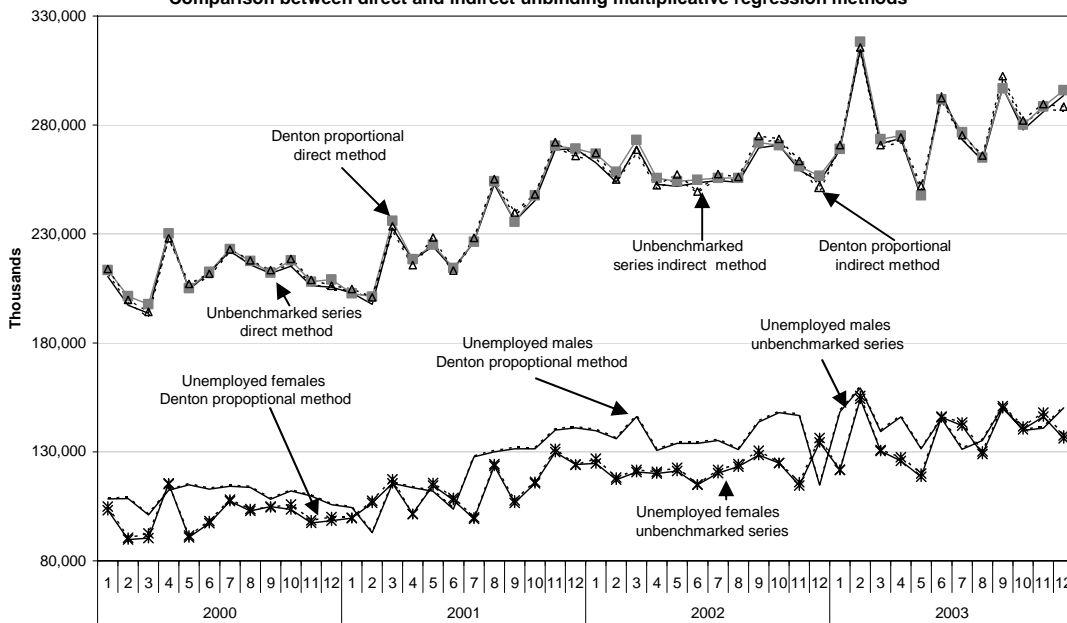
Diagram 7 and Diagram 8 display the total unemployed persons, and its component series: unemployed males and unemployed females, for 2000-2003. The composite series is derived from direct and indirect benchmarking techniques. In Diagram 7 all component series are benchmarked using the Denton proportional method whereas in Diagram 8 the benchmarking is achieved using the unbinding regression method. Note that all series are seasonally adjusted by direct and indirect seasonal adjustment method.

First, let us mention that all benchmarking methods displayed in Diagram 7 and 8 seem to correct the bias in the monthly series. Second, there are no significant differences between the composite seasonally adjusted series from direct and indirect seasonal adjustment methods. Also it is shown that the composite seasonally adjusted series obtained from the Denton proportional benchmarking method and the unbinding multiplicative regression method are very close to each other. It can be seen that the seasonally adjusted series is similar in smoothness for the two benchmarking methods mentioned above, and the amount of revisions in the seasonally adjusted composite data is small (the movement preservation statistic is 0.5285 for Denton proportional and 0.4359 for unbinding multiplicative regression

**Diagram 7: Total unemployed persons - seasonally adjusted series  
Comparison between direct and indirect Denton proportional methods**



**Diagram 8: Total unemployed persons - seasonally adjusted series  
Comparison between direct and indirect unbinding multiplicative regression methods**



methods, and the smoothness diagnostic for the seasonally adjusted series is 6.6106 and 6.7680, respectively).

## 6. Discussion

The Israel labour force series was chosen because of urgent need to publish the aggregate monthly figures. It is worthwhile to extend this study to series with significant seasonal fluctuations and Jewish moving festival and trading day effects.

## 6.1 Choosing between benchmarking models

In terms of moving preservation, the proportional Denton approach seems to be the superior binding method, as shown by IMF staff (2002). However, from the seasonal adjustment point of view, using other binding models can be desirable for series with intermediate or high seasonality, in particular, the binding regression model with an ARMA model for the errors  $e_t$ . There are two main reasons for this: a) the standard deviations of the estimated benchmarked series are smaller for this method; b) the ARMA modeling of the errors  $e_t$  may improve the benchmarked figures and may lead to the preservation of seasonal movements in the series. Improvement in the estimates of the benchmarked series is not surprising as more information about the data is explored. Obviously, superiority of a specific regression model (additive, multiplicative or mixed) depends on the original data and the appropriate model must be chosen after suitable data analysis.

As seen in the Israel labour force data, choosing the benchmarking model is independent of selecting the seasonal decomposition model for seasonal adjustment. In our study, the most appropriate benchmarking model for binding regression method is the additive model, although the multiplicative seasonal model is used for all these series. Therefore, one can treat the benchmarking and the seasonal adjustment as two separate processes.

For employed females series with very low seasonality, an ARMA model for errors does not improve significantly the estimation of the benchmarked series. Moreover, it can change the seasonal pattern and increase the contribution of the irregular component in the series. The significant reduction of standard deviation in the binding regression methods may indicate overfitting of the model, when artificial smoothing of the data may cause distortion of the estimation of prior adjustment and seasonal factors.

The unbinding regression methods have the best movement and seasonal pattern preservation properties among all analyzed methods. As the two analyzed Denton models, these methods provide the benchmarked estimates without significant revisions to the irregular component of the time series, which can avoid the danger of affecting the estimation of the prior adjustment factors.

## 6.2 Indirect method for benchmarking aggregate series

Diagrams 7 and 8 present the composite series provided by direct and indirect benchmarking methods and the results are very similar. Naturally, the accounting links are preserved through the indirect benchmarking, hence this method with good movement preservation properties may be the preferable one. The advantages of the indirect benchmarking method exist when the chosen benchmarking model preserves month-to-month movements and the seasonal pattern in the sub-series. Moreover, using the indirect benchmarking method matches the concept of indirect seasonal adjustment, and for the composite series for which indirect seasonal adjustment method is applied the indirect benchmarking method may be suitable. The motivation to use the indirect method in benchmarking is the same as in seasonal adjustment; it allows an easier explanation of the changes in the aggregate data through the changes in the sub-series. It must be pointed out that the indirect seasonal adjustment approach is not applied to all composite series. In this case the direct benchmarking methods, such as Di Fonzo and Marini (2003) technique, or the regression method in section 5 may be more appropriate.

### 6.3 Quality issues

The quality concepts used in statistical organizations have changed during the last decades. Nowadays, several statistical agencies have adopted Eurostat quality concept defined by several criteria Linden (2001) and Haworth and others (2001)). Here we refer only to the criteria that are relevant for benchmarking.

1. **Relevance:** a statistical product is relevant if it meets users' needs. Benchmarking process meets this criterium because there is a strong demand by the users (e.g. central banks, economic organizations, universities) for reliable high-frequency data about socio-economic phenomena that matches the low-frequency figures. The necessity of seasonally adjusted data forces us to focus on the benchmarking model with the desired qualities for seasonal adjustment.
2. **Accuracy** is defined as the closeness between the estimated value and the unknown true population value. Applying the benchmarking procedure we assume that the low-frequency data is more reliable either in terms of standard deviations, or in terms of closeness to the true figures. This is especially important for administrative data such as national accounts series.

- 3. Coherence:** The coherence between statistics is oriented towards the comparison of different statistics, which are generally produced in different ways and for different primary uses. The messages that statistical agencies convey to users will clearly relate to each other, or at least not contradict each other. The benchmarking procedure meets this criterium, because by definition it combines data from two several sources with different frequencies in order to minimize the existing discrepancies. The indirect benchmarking method allows adjusting the composite series through the sub-series without violating accounting constraints in the system of time series. By doing so one can avoid discrepancies and additional revisions in the original data by applying the benchmarking procedure  $n-1$  times for the system of  $n$  time series.

## 7. Conclusions

In this paper the properties of the benchmarking procedures are analyzed with a special emphasis on seasonal adjustment and quality issues. As mentioned in section 7.3, the benchmarking problem meets the statistical quality criteria that were proposed by the Eurostat and other international statistical organizations: relevance, accuracy and coherence of the statistical data.

Three benchmarking approaches are considered: the binding regression method with ARMA model for errors, the Denton method and the unbinding regression method with ARMA model for errors. The Denton proportional method seems to be the preferable one among the binding benchmarking methods from the seasonal pattern preservation point of view. Nevertheless, other benchmarking approaches that preserve seasonal pattern of the series can be considered. In section 5.3 it was shown that the binding regression approach may be appropriate for the series with intermediate and high range of seasonality. For the benchmarking of survey data, where the survey errors are known, the unbinding regression approach may be useful. This method provides the best diagnostics for movement and seasonal pattern preservation.

The benchmarking model for a system of time series, based on the regression approach, and the indirect benchmarking method for the composite series are introduced. The direct and the indirect benchmarking methods provide very close

composite benchmarked series, for the labour force series. The indirect approach is preferable especially for series that are seasonally adjusted by indirect method in order to preserve its advantages. However, for composite series that is seasonally adjusted by the direct method, the direct benchmarking method using the Denton or regression approach may be preferable.

## 8. References

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