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## **Benchmarking Techniques in the Spanish Quarterly National Accounts**

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### **Abstract**

In this paper I review some of the benchmarking techniques used to compile the Spanish Quarterly National Accounts (QNA). The basic framework has three main stages. In the first one, univariate and multivariate temporal disaggregation procedures are used to generate raw data. The second stage consists on the seasonal and calendar adjustment of the raw data, by means of the ARIMA model-based methodology as implemented in the TRAMO-SEATS programs. Finally, in the third stage, the unbalanced seasonally and calendar adjusted data are transformed in order to ensure transversal constraints (e.g., accounting identities) as well as temporal consistency (e.g., quantitative coherence with the annual data provided by the National Accounts). This last stage is fulfilled using multivariate temporal disaggregation procedures and purely transversal adjustment methods. In the last section of the paper, I will examine some issues related to the special requirements of the flash estimates of the Gross Domestic Product (GDP), the implementation of the chain-linking methodology to estimate volume data, technical improvements of the temporal disaggregation procedures, and compilation of estimates through quarterly supply-use tables (the QSU model).

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## 1. INTRODUCTION

The Spanish Quarterly National Accounts (QNA) is a statistical operation of a synthetic nature in the sense that it transforms the information provided by a wide set<sup>2</sup> of short-term monthly and quarterly indicators into a coherent, quantitatively consistent estimation of the aggregate economic activity, which is elaborated from three different perspectives: supply (or production), demand (or expenditure) and income generation. These estimates conform to the same principles of coherence and equilibrium as the Spanish Annual National Accounts (ANA) and therefore also to the European System of National Accounts of 1995 (ESA-95). Furthermore, the Spanish QNA includes estimates for the level of employment in ESA-95 terms, as well as the accounts for the institutional sector Rest of the World, see Eurostat (1998) for a complete analysis of QNA.

Therefore, due to its intrinsic characteristics, the Spanish QNA is the result of a complex, multidimensional benchmarking exercise and its reliance on formal, econometrically explicit benchmarking techniques is a logical consequence. But not only formal or theoretical reasons move the Spanish QNA to adopt these techniques. Operational and practical considerations related to strict time schedules can only be fulfilled by means of a heavily computer-based production environment. In this production mode, formal models are instrumental because they can be easily transformed in efficient computer code. Computerization of the procedures also ensures safety, data integrity, fast processing and reproducibility.

This reliance will likely continue in the future as more ambitious and sophisticated procedures (e.g., chain-linking, QSU) are envisaged, basic short-term indicators become more diffuse as a result of an ever-changing economic system and the relevance of explicit measures of the uncertainty that surrounds the estimates are demanded (e.g., to properly calibrate QSU models).

This paper is structured as follows. In the second section I describe the univariate and multivariate temporal disaggregation procedures that are used to generate the raw data. In the third section, the benchmarking methods used to compile the calendar and seasonally adjusted data are put forward. The fourth section deals with some future developments, which have varying degrees of maturity. Computational details are briefly commented in the fifth section. The paper includes an appendix that describes a tentative, preliminary design of a QSU model strongly dependent on benchmarking techniques.

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<sup>2</sup> Around 1200 time series are processed in the current system of the Spanish QNA.

## 2. RAW DATA COMPILATION

In this section I present a simplified but (I hope) complete model that incorporates the main features of the compilation practices used in the Spanish QNA. In order to better substantiate the model, I will concentrate it on the estimation of the supply-side and demand-side components of GDP, at current and constant prices. First, I will describe the derivation of the raw (or unadjusted) data as a combination of a low-frequency input, the data provided by the Annual National Accounts (ANA), and a high frequency input, monthly or quarterly short-term indicators that approximate the unobserved subannual evolution of the ANA variables. The ANA data are considered the benchmark and high-frequency indicators are properly transformed to match quantitatively this benchmark.

Raw data compilation comprises three elements: estimation of supply-side components of GDP by means of univariate temporal disaggregation procedures, computation of GDP as the transversal (i.e., quarterly) aggregation of its previously estimated supply-side constituents, and, finally, estimation of the demand-side components of GDP using multivariate temporal disaggregation methods that take into account the supply-derived quarterly GDP.

### 2.1. General information set

The model requires as input the following elements:

- Annual data

Let  $Y = \{Y_{j,T} : j = 1..m, m+1, ..M; T = 1..N\}$  be the observed ANA series that play the role of quantitative benchmarks of the system. The cross-section index  $j$  takes in the  $m$  supply-side components of GDP as well as the  $M-m$  demand-side components of GDP.

GDP itself is a derivative formed as the transversal aggregation of the elements of  $Y$ :

$$[2.1] \quad Z = \begin{bmatrix} i_m & 0_{M-m} \end{bmatrix}' Y = \begin{bmatrix} 0_m & i_{M-m} \end{bmatrix}' Y$$

An immediate implication of [2.1] is:

$$[2.2] \quad \begin{bmatrix} i_m & -i_{M-m} \end{bmatrix}' Y = EY = 0$$

## Remark

- Additional constraints may be considered by simply expanding columnwise the matrix  $E$  to include them. Although this possibility seems trivial from a theoretical point of view, it poses severe computational problems when it is combined with the temporal constraints that are a hallmark of the QNA. These problems are further discussed in the paragraphs devoted to the QSU model.
- High frequency indicators

Let  $x = \{x_{j,t} : j = 1..m, m+1,..M; t = 1..n\}$  be the observed high-frequency (e.g., monthly,  $s=12$ , or quarterly,  $s=4$ ) indicators that operate as subannual proxies for the ANA aggregates  $Y$ . The indicators should be chosen in order to mimic the ANA compilation practices and they have to show a high degree of low-frequency conformity with the ANA time series. This task is quite delicate and, as in any standard econometric exercise, should combine a detailed knowledge of the available short-term indicators (sampling procedures, methodological issues, statistical properties, etc.) and ANA estimation procedures.

The temporal dimension of  $x$  should cover, at least, the (implied) high frequency dimension of  $Y$ :  $n \geq sN$ .

## Remarks

- To simplify the exposition, I assume that there is only one indicator  $x_j$  for each annual variable  $Y_j$ . The model remains basically unchanged if I assume that there are several indicators for each annual variable, so  $x_j$  becomes a matrix instead of a vector.
- Despite this theoretical possibility, the one-indicator case is the most frequently used in real applications. This case avoids some estimation problems and eases the control and analysis of the models. Therefore, if  $p_j$  is the number of indicators<sup>3</sup> linked to  $Y_j$  such as  $p_j > 1$ , a synthetic or combined indicator may be formed as:

$$[2.3] \quad x_{j,t} = \sum_{h=1}^{p_j} w_{j,h,t} x_{j,h,t}$$

- In the QNA, the weights  $w$  are derived from the ANA data (e.g., its SU table) and may be time-varying (e.g., if a chain-linking methodology is used).
- Very often, as a useful statistical validation tool, the structure  $w$  used in [2.3] is compared with the one generated by a (static) factor analysis of the time series  $x_{j,h,t}$ .

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<sup>3</sup> Excluding intercepts.

## 2.2. Supply-side components of GDP

I will review the main specific elements related to the estimation of the supply-side components of GDP.

### 2.2.1. High-frequency model

In order to perform the estimation of QNA I assume that there is a linear model that links the (observable) high-frequency indicators with the (unobservable) QNA variable. The model assumes a static relationship between  $y$  and  $x$ , additively perturbed:

$$[2.4] \quad y_j = x_j \beta_j + u_j \quad j = 1..m$$

The innovation  $u$  follows a stationary AR(1) process (therefore, assuming cointegration between  $y$  and  $x$ ) or a non-stationary I(1), random walk process (therefore, excluding cointegration between  $y$  and  $x$ ):

$$[2.5] \quad \begin{aligned} u_j &\sim N(0, v_j) \\ v_j &= \sigma_j^2 [(I_n + \rho_j \Xi)' (I_n + \rho_j \Xi)]^{-1} \quad -1 < \rho_j \leq 1 \end{aligned}$$

The auxiliary matrix  $\Xi$  is defined as:

$$\Xi = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ -1 & 0 & \dots & 0 & 0 \\ 0 & -1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -1 & 0 \end{bmatrix}$$

Initial conditons depend on the value of  $\rho_j$ :

$$[2.6] \quad u_{j,0} = \begin{cases} N(0, \sigma_j^2 / (1 - \rho_j^2)) & \text{if } -1 < \rho_j < 1 \\ 0 & \text{if } \rho_j = 1 \end{cases}$$

The model includes a temporal constraint that makes  $y_j$  quantitatively consistent with its annual counterpart  $Y_j$ :

$$[2.7] \quad Y_j = C y_j$$

with a temporal aggregation-extrapolation matrix defined as:

$$[2.8] \quad C = (I_N \otimes c / O_{N, n-sN})$$

where  $N$  is the number of low-frequency observations,  $\otimes$  stands for the Kronecker product,  $c$  is a row vector of size  $s$  which defines the type of temporal aggregation and  $s$  is the number of high frequency data points for each low frequency data point. If  $c=[1, 1, \dots, 1]$  we would be in the case of the temporal aggregation of a flow, if  $c=[1/s, 1/s, \dots, 1/s]$  in the case of the average of an index and, if  $c=[0, 0, \dots, 1]$ , an interpolation would be obtained.

When the size of the indicators' sample is not conformable with the one of the low-frequency series ( $n > sN$ ), a situation of extrapolation shows up. In this case, the problem can easily be solved by simply extending the temporal aggregation matrix by considering new columns of zeroes which do not distort the temporal aggregation relationship and that do not pose any difficulty to the inclusion of the last  $n-sN$  data points of the indicators in the estimation process of  $y$ .

### Remarks

- What is called "direct estimation" is a special case of [2.4]-[2.5] that arises when  $\rho_j=1$ ,  $\beta_j=1$  and  $\sigma_j=0$ . Therefore, model-based procedures encompass (and provide a test for) the direct estimation case.
- The development of dynamic models that generalizes or encompasses [2.4]-[2.6] is an active research field, see the last section of the paper.
- Improvements in the treatment of the initial conditions are also being actively investigated, see Proietti (2004) and Di Fonzo (2005).

### 2.2.2. Estimation

The estimation of  $y_j$  according to the model [2.4]-[2.6] and satisfying the constraint [2.7]-[2.8] is performed by means of the Chow-Lin (1971) or Fernández (1981) procedures. The proposed Best<sup>4</sup> Linear Unbiased Estimator (BLUE) estimator adopts the form:

$$[2.9] \quad \hat{y}_j = x_j \hat{\beta}_j + v_j C' V_j^{-1} \hat{U}_j = x_j \hat{\beta}_j + L_j \hat{U}_j$$

with  $V_j = C' v_j C$ . The annual disturbance term is defined as:

$$[2.10] \quad \hat{U}_j = Y_j - X_j \hat{\beta}_j$$

with  $X_j = C x_j$ .

Equation [2.9] expresses the estimator as the combination of a term linearly linked to the indicator<sup>5</sup> and a temporally disaggregated residual series. The main feature of the estimator is the dependency of the temporal disaggregation filter  $L_j$  on the form adopted by the quarterly model, and in particular, on the

<sup>4</sup> In the sense of minimizing the mean squared error.

<sup>5</sup> Sometimes,  $x_j \hat{\beta}_j$  is called "scaled indicator".

dynamic structure of its disturbance term  $u_j$ . The Generalized Least Squares (GLS) estimator of  $\beta_j$  is:

$$[2.11] \quad \hat{\beta}_j = (X_j' V_j^{-1} X_j)^{-1} (X_j' V_j^{-1} Y_j)$$

An important advantage of this method is that it generates confidence intervals for the quarterly estimates from the corresponding variance-covariance matrix:

$$[2.12] \quad \Sigma_{\hat{y}_j} = (I_n - L_j C) v_j + (x_j - L_j X_j) \Sigma_{\hat{\beta}_j} (x_j - L_j X_j)'$$

The last equation implies that the uncertainty associated with the quarterly estimates is tied to two sources: one related to the variability of the quarterly stochastic disturbance term  $u$  and the other linked to the imprecision in which we incur when estimating  $\beta_j$ .

Expressions [2.9] to [2.12], which fully define the Chow-Lin method, require for its implementation prior knowledge of the variance-covariance matrix  $v_j$  of the quarterly disturbance term  $u_j$  which depends on  $\rho_j$ , see [2.5]. The estimation of this parameter is accomplished by means of the evaluation of the implied log-likelihood function of the low-frequency model. The function is (dropping subindex  $j$ ):

$$[2.13] \quad \begin{aligned} \ell(\beta, \sigma_a^2 | \bar{\rho}) = & -\frac{N}{2} \ln(2\pi\sigma^2) - \frac{1}{2} \ln(|Cv(\bar{\rho})C'|) - \\ & - \frac{1}{2\sigma^2} (Y - X\beta)' (Cv(\bar{\rho})C')^{-1} (Y - X\beta) \end{aligned}$$

This optimization is performed by means of a grid search on the stationary domain of  $\rho$ , and pinning down the values of  $\beta_j$  and  $\sigma_j^2$  that maximizes [2.13] conditioned on the selected value for  $\rho_j$ , see Bournay and Laroque (1979).

## Remarks

- Sometimes, the likelihood profile is rather flat and the analyst has to solve the lack of discrimination that the sample exerts on the alternative (and admissible) values for  $\rho_j$  by proper selection of its value, according to non-sample information.
- The case  $-1 < \rho_j \leq 0$  poses serious estimation problems linked to an unstable relationship between  $y_j$  and  $x_j$  that should be addressed by redefining the set of high frequency indicators and/or setting  $\rho_j$  at a small, positive value (e.g., 0.25).
- Little difference arises between Chow-Lin and Fernández estimates if  $\rho_j \geq 0.8$ . Consequently, if  $\rho_j$  exceeds an upper bound (e.g., 0.98) the Fernández procedure is directly applied (dropping subindex  $j$ ):

$$\hat{\beta} = \left[ x' C' [C(D'D)^{-1} C']^{-1} Cx \right]^{-1} x' C' [C(D'D)^{-1} C'] Y$$

$$\hat{y} = x\hat{\beta} + (D'D)^{-1} C' [C(D'D)^{-1} C']^{-1} (Y - Cx\hat{\beta})$$

$$D = I_n + \mathcal{E}$$

- Another interesting feature of the Fernández procedure is that it generates less revisions than the Chow-Lin method in the estimated  $y$  when the annual time series  $Y$  is expanded. This may be due to the more simple nature of its estimation and the stability of the  $L$  filter, see Proietti (2004).
- The variances of the estimates, see equation [2.12], play an important role in the assesment of its uncertainty and as a critical input in the balancing procedures.
- The model and the estimation procedure may be easily extended to include a richer dynamic structure for the innovations. This is the case for the method proposed by Litterman (1983). This author assumes an ARIMA(1,1,0) model for the innovations instead of the ARIMA(1,0,0) proposed by Chow-Lin or the ARIMA(0,1,0) proposed by Fernández. In this case the high-frequency variance-covariance matrix becomes:

$$v_j = \sigma_j^2 \left[ (I_n + \rho_j \mathcal{E})' (I_n + \mathcal{E})' (I_n + \mathcal{E}) (I_n + \rho_j \mathcal{E}) \right]^{-1} \quad -1 < \rho_j < 1$$

and the initial conditions are:

$$u_{j,0} = \Delta u_{j,0} = 0$$

The Litterman method is theoretically attractive since it encompasses the Fernández model and tries to generate series minimizing a linear combination of the volatilities of the period-to-period growth rates and the period-to-period accelerations, see Pinheiro and Coimbra (1993). However, the empirical and Monte Carlo evidence show that its performance is sometimes disappointing. This fact is due to the flatness of the implied likelihood profile and, therefore, the corresponding observational equivalence in a wide range of values for  $\rho_j$  (e.g., [-0.6 - 0.7]).

- Barbone et al. (1981) propose to use the following Weighted Least Squares (WLS) objective function instead of [2.13]:

$$\psi(\beta, \sigma_a^2 | \bar{\rho}) = \frac{1}{2\sigma^2} (Y - X\beta)' (C\bar{\rho}C')^{-1} (Y - X\beta)$$

An easy and sensitive robustness check is to compare the WLS estimates with the (maximum likelihood) estimates derived from [2.13].

### 2.3. GDP estimation

GDP is simply obtained as the sum of these supply-side variables:

$$[2.14] \quad \hat{z} = \sum_{j=1}^m \hat{y}_j$$

#### Remark

- The supply-side nature of the GDP estimation should be considered only as a first-round step in a sequential estimation process. Independent and simultaneous estimation of the demand-side components of GDP are used to ensure overall consistency of both estimates. More on this in the paragraph devoted to QSU.

### 2.3. Demand-side components of GDP

The estimation of the quarterly demand-side, raw data components of GDP is performed taking into account simultaneously the intrayear information provided by the short-term indicators and the contemporaneous high-frequency constraints that bind the estimations. The method applied is the one developed by Di Fonzo (1987, 1990, 1994). Next, I will review the model and the procedures used.

#### 2.3.1. High-frequency model

The high-frequency model belongs to the Seemingly Unrelated Regression Equations (SURE) class:

$$[2.15a] \quad \underbrace{\begin{bmatrix} y_{m+1} \\ y_{m+2} \\ \dots \\ y_M \end{bmatrix}}_y = \underbrace{\begin{bmatrix} x_{m+1} & 0 & \dots & 0 \\ 0 & x_{m+2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & x_M \end{bmatrix}}_x \underbrace{\begin{bmatrix} \beta_{m+1} \\ \beta_{m+2} \\ \dots \\ \beta_M \end{bmatrix}}_\beta + \underbrace{\begin{bmatrix} u_{m+1} \\ u_{m+2} \\ \dots \\ u_M \end{bmatrix}}_u$$

Expressing [2.15a] in compact form:

$$[2.15b] \quad y = x\beta + u$$

The innovations are distributed according to:

$$[2.16] \quad u \sim N(0, \Sigma \otimes (\Psi' \Psi)^{-1})$$

being  $\Sigma$  a matrix that captures the contemporaneous variances and covariances of the disturbances of the model and  $\Psi$  is a matrix that depends on the dynamic structure of  $u$ . Usual parameterizations of  $\Psi$  are  $\Psi=I$  (multivariate

white noise) and  $\Psi=I+\mathcal{E}$  (multivariate random walk). In the second case, initial conditions are  $u_0=0$ .

The (unobservable)  $y=\{y_{j,t} : j=m+1..M, t=1..n\}$  must satisfy two constraints, one longitudinal:

$$[2.17] \quad Cy_j = Y_j \quad \forall j$$

and another of a transversal nature:

$$[2.18] \quad \sum_{j=m+1}^M y_j = \hat{z}$$

where  $C$  is the aforementioned temporal aggregation matrix and  $\hat{z}$  is the supply-side estimated GDP, see [2.14].

Note that, in the Di Fonzo method, (the estimate of)  $z$  is part of the set of available information and, therefore, is not itself subjected to additional estimation. The general structure of the problem is shown in the following illustrative table:

**Table 2.1: Quarterly variables subject to temporal and transversal constraints**

Year	Quarter	Series				Total
		$y_1$	$Y_1$	$y_2$	$Y_2$	$z$
	1	$y_{1,1,1}$		$y_{2,1,1}$		$\mathbf{z}_{2,1,1}$
	2	$y_{1,2,1}$		$y_{2,2,1}$		$\mathbf{z}_{2,2,1}$
	3	$y_{1,3,1}$		$y_{2,3,1}$		$\mathbf{z}_{2,3,1}$
	4	$y_{1,4,1}$		$y_{2,4,1}$		$\mathbf{z}_{2,4,1}$
	1		$\mathbf{Y}_{1,1}$		$\mathbf{Y}_{2,1}$	
	1	$y_{1,1,2}$		$y_{2,1,2}$		$\mathbf{z}_{2,1,2}$
	2	$y_{1,2,2}$		$y_{2,2,2}$		$\mathbf{z}_{2,2,2}$
	3					
	4					
	2					

The values on this table are twofold: the ones in **black** stand for the exogenous data of the problem (2 annual totals and 6 quarterly totals), while the ones in *cursive* reflect the estimations to be carried out (12 quarterly data points). This estimation structure is termed “conditional extrapolation”, see Di Fonzo (1990, 1994) for a throughout analysis.

Expressing the constraints [2.17] and [2.18] using matrix notation we have:

$$[2.19] \quad Hy = Y_e$$

being:

$$[2.20] \quad H = \begin{bmatrix} i'_{M-m} \otimes I_n \\ I_{M-m} \otimes C \end{bmatrix} \quad \text{and} \quad Y_e = \begin{bmatrix} \hat{z} \\ Y \end{bmatrix}$$

### 2.4.2. Estimation

The BLUE estimator of  $y$  according to [2.15]-[2.16] and verifying [2.19]-[2.20] is provided by:

$$[2.21] \quad \hat{y} = x\hat{\beta} + L(Y_e - X_e\hat{\beta})$$

being  $\hat{\beta}$  the estimate using generalized least squares in a SURE context:

$$[2.22] \quad \hat{\beta} = (X_e' V_e^{-1} X_e)^{-1} (X_e' V_e^{-1} Y_e) \quad \text{and} \quad X_e = Hx$$

$L$  is the distribution filter of the residual series:

$$[2.23] \quad L = vH' V_e^{-1}$$

The interpretation of these results is, essentially, the same as the one derived from the examination of the results of the Chow-Lin procedure. There exists only one technical specificity which is closely related to the nature of the matrix containing the constraints,  $H$ . As this matrix is not a full rank one, its inversion requires the use of a generalized inverse like the Moore-Penrose inverse:

$$[2.24] \quad H^{-} = (H' H)^{-1} H'$$

### Remarks

- The Di Fonzo procedure is quite flexible, since it allows the implementation of pure disaggregation, unconditional extrapolation and conditional extrapolation. As it has been already mentioned, the last case is critical from an operational point of view.
- Dynamic generalization of the model [2.15] have been tried out replacing the uncoupled multivariate random walk innovations by uncoupled VAR(1) innovations. The results showed little difference between the estimates of both models but a huge increase in the computing time that penalized the VAR(1) parameterization. The similarity may be due to the above mentioned convergence of results of the Chow-Lin and Fernández methods when the  $\rho$  parameter is close to one.
- Comparison between the estimates derived in an univariate framework (e.g., using the Fernández method) and those derived in a multivariate plus transversally constrained framework is a useful check to gauge the impact of the balancing constraints.

### 3. SEASONAL AND CALENDAR ADJUSTED DATA COMPILATION

Quarterly aggregates ( $\hat{y}_j$ ,  $j=1..m$ ,  $m+1..M$ ) and GDP ( $\hat{z}$ ) are corrected from seasonal and calendar variation by means of an ARIMA model-based procedure, as it has been implemented in the TRAMO-SEATS programs, see Gómez and Maravall (1994, 1996) and Caporello and Maravall (2004).

GDP is seasonal and calendar adjusted directly, i.e. adjusting raw GDP data rather than deriving it as the sum of its seasonal and calendar adjusted constituents. Direct vs indirect adjustment of derived series is an open issue<sup>6</sup> but the direct approach fits better than the indirect one in the general framework used by the Spanish QNA, see INE (2002) for a detailed exposition.

These new series do not verify neither the temporal constraint that relates the quarterly estimates to their annual counterparts nor the transversal restriction that equates GDP with their supply-side and demand-side components.

The procedure applied in order to solve both inconsistencies is a multivariate extension of the Denton (1971) method, see Di Fonzo (1994) and Di Fonzo and Marini (2003). Formally, it can be described as follows.

Let  $\bar{y} = \{\bar{y}_j^{sac} : j=1..m, t=1..n\}$  be a matrix  $n \times m$  denoting the available observations of  $m$  quarterly time series which do not comply with the constraints that generate  $Y_e$ , see [2.19] to [2.20].

The balanced and temporally consistent time series are the output from the following constrained quadratic optimization program:

$$[3.1] \quad \underset{\tilde{y}}{MIN} \quad (\tilde{y} - \bar{y})' D' D (\tilde{y} - \bar{y}) \quad s.t. \quad H\tilde{y} = Y_e$$

In the program [3.1] the objective function reflects the volatility of the discrepancies between the quarter-to-quarter growth rates of the balanced series and those of the unbalanced ones. After some mathematical manipulation, an explicit expression can be derived:

$$[3.2] \quad \tilde{y} = \bar{y} + (D' D)^{-1} H' [H (D' D)^{-1} H']^{-1} (Y_e - H\bar{y})$$

The interpretation of equation [3.2] is fairly simple: the quarterly balanced series are the result of adding up a correction factor to the unbalanced series. This correction factor originates from the distribution of the discrepancy between the preliminary unbalanced estimates and the constraint series  $Y_e$ .

Extrapolated quarters (those that do not have to satisfy an annual constraint) are estimated through the van der Ploeg (1982) balancing method:

<sup>6</sup> See Gómez (2000), Planas and Campolongo (2000), Maravall (2002), and the references cited therein.

$$[3.3] \quad \hat{y} = \tilde{y} - \Sigma A' [A \Sigma A']^{-1} A \tilde{y}$$

$$\text{with } \hat{y} = \begin{bmatrix} \tilde{y} \\ \hat{z} \end{bmatrix} \quad \tilde{y} = \begin{bmatrix} \bar{y} \\ \hat{z} \end{bmatrix} \quad A = \begin{bmatrix} -i'_m \\ I \end{bmatrix}$$

The variance-covariance matrix  $\Sigma$  is considered diagonal and their values are derived from the variances provided by the temporal disaggregation procedures outlined in the second section. Of course, the element  $\Sigma(m+1, m+1) = 0$ , i.e., GDP estimates are not modified since they are the basic constraint.

The interpretation of this equation is straightforward: the vector of balanced variables is the result of adjusting the preliminary estimations according to the observed discrepancy, taking into account the variance-covariance structure of the preliminary estimates. The similarity of this solution with the one offered by the multivariate Denton method is due to their common linear-quadratic approach to benchmarking.

When the process has been applied to the  $m$  supply-side components of GDP it is repeated to the  $M-m$  remaining demand-side components of GDP.

### Remarks

- The proper combination of temporal disaggregation and seasonal adjustment is an open issue. More on this in the last section.
- The van der Ploeg (1982) procedure shows some interesting properties. The magnitude of revisions, in absolute value, is an increasing function of the variance of the initial estimate ( $\sigma_{jj}$ ), i.e., the higher the uncertainty associated with the initial estimate, the larger the size of its revision.
- If we can consider that a determined preliminary estimation can be pinned down to a concrete value ( $\sigma_{jj} = 0$ ), then there will not be any adjustment:  $\hat{y}_j = \tilde{y}_j$
- The diagonal assumption is made to simplify the calibration of the procedure, not for substantive matters. If the uncertainty in the estimation of two variables evolves towards the same direction ( $\sigma_{ij} > 0$ ), its revisions will also take that direction. If, on the contrary, they have a negative covariation the adjustments will go in opposite directions.
- The covariances are, due to their nature, more difficult to calibrate. Usually, its calibration may be resorted to an indirect procedure which is based on the historical correlation between the variables. In this case, once the variances have been estimated, the covariances are derived according to  $\sigma_{ij} = \rho_{ij} \sqrt{\sigma_{ii} \sigma_{jj}}$ . More sophisticated procedures may be used, akin to those used in the field of asset management, see Litterman and Winkelmann (1998).

## **4. NEW DEVELOPMENTS**

The basic Spanish QNA system is currently being revised in several key aspects due to: (i) the introduction of flash estimates, (ii) the chain-linking methodology to compile volume measures, (iii) technical developments concerning temporal disaggregation methods, and (iv) compilation of estimates through quarterly supply-use tables (the QSU model). I will briefly review the main issues related to them.

### **4.1. Flash estimates**

Flash estimates are an attempt to increase the timeliness (or opportunity) of the QNA. Recently, INE has introduced this estimate for GDP in order to expand the set of available short-term indicators, see INE (2005a).

As a general rule, the procedure employed for the compilation of the flash estimate is the same as the one used for the quarterly standard estimate. The major differences are the preferential use of monthly indicators instead of quarterly ones, a higher resort to forecasting techniques and a more pronounced weight of supplementary assessment elements (e.g., BVAR models to check the extrapolations).

Although the recommended practice is to rely on the same system to compile the flash and the standard versions of the QNA, the fact is that flash estimates pose a specific challenge to the practice of QNA, from an operational perspective as well as from an informational perspective. The operational aspect of the challenge is related to the need to estimate accurately the complete system more rapidly. Improved computer codes, efficient databases and well-defined working schedules are now (even) more important than ever.

The informational challenge is also quite demanding. Flash estimates require the use of monthly instead of quarterly indicators and bring to the scene the convenience of combining explicit dynamic models for the indicators with the short-term evolution of the QNA time series. One possibility is to design a system that generates monthly estimates of the QNA variables using bivariate models, see Casals et al. (2004) for an interesting approach.

### **4.2. Introduction of the chain-linking methodology**

According to the recommendations expressed by Eurostat (2004), INE will introduce the chain-linking methodology to compile the volume measures of its QNA, see INE (2005b). The main elements of the chain-linking methodology are:

- Index formula: Laspeyres index for quantities and Paasche index for prices.
- Weights structure: Annual weights, corresponding to year  $T-1$ .

- Chaining method: Annual overlap method<sup>7</sup>. Possible discontinuities between fourth quarter of year  $T$  and first quarter of year  $T+1$  will be solved by means of Denton procedure.
- Seasonal and calendar adjustment: Chain-linked indexes will be corrected from calendar and seasonal effects in their original frequency.
- Temporal disaggregation: Raw and adjusted chain-linked indexes will be used as high-frequency inputs for the temporal disaggregation methods. The low-frequency input will be the corresponding chain-linked ANA data.

Aggregates valued at current prices are additive and the basic methodology outlined in the paper remains unchanged. However, volume-measured aggregates are not additive and the role of transversal constraints becomes redundant. Temporal consistency will be ensured by means of the annual overlap method and the standard temporal disaggregation methods described in this paper.

### 4.3. Technical improvements in temporal disaggregation methods

Several authors have proposed the use of dynamic models to perform temporal disaggregation, see Gregoir (1994), Salazar et al. (1994), Santos and Cardoso (2001), Di Fonzo (2003), Proietti (2004), among others. These models generalize and sometimes encompass the standard Chow-Lin model (including its extensions due to Fernández and Litterman), are more akin to the standard econometric practice in the modeling of high-frequency time series, and may provide more robust (i.e., more stable) results. Currently, the Santos-Cardoso method is being carefully studied.

Another issue is the proper combination of seasonal adjustment and temporal disaggregation, see Eurostat and European Central Bank (2002). The current system considers a sequence where temporal disaggregation is first applied and seasonal adjustment is performed on the resulting series. Very likely, the new system will change the ordering. The main reasons are:

- Better seasonal adjustment may be performed if it is done at the native frequency of the indicators.
- Calendar effects, if present, are also better modelled and corrected at monthly frequency (if possible).
- Seasonal components are unobservable at the annual frequency, so their role in the compilation of seasonally adjusted temporal disaggregates should be explicitly controlled.

### 4.4. Compilation via QSU model

The use of the so-called QSU model (Australian Bureau of Statistics, 2004) to compile or validate initial QNA estimates is currently being investigated. The possibility to check those estimates via an equilibrium model that includes at

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<sup>7</sup> See Bloem et al. (2001) for a throughout analysis of the different overlapping methods.

the same time the generation of resources and its allocation among competitive uses is quite interesting, both from a statistical point of view and as a check on the overall accuracy of the estimates.

The use of QSU implies a significative increase in the number of transversal constraints usually considered in the Spanish QNA. Therefore, the combination of these constraints with the standard temporal restrictions poses severe computational problems due to the overwhelming dimension of the corresponding  $H$  matrices (see [2.20]), rendering it impractical. Therefore, the QSU model should be limited to current quarters (i.e., when the annual constraint is not binding).

Notwithstanding its appeal, some important issues should be addressed: the proper role of independent statistical sources to estimate intermediate consumption, the treatment of seasonality and the issue of the non-additivity of chain-linked volume measures. From a technical point of view, the procedure of van der Ploeg (1982) plays an important role in the balancing process, see Appendix A for additional details.

## 5. COMPUTATIONAL ISSUES

Computational and software tools are a key ingredient in modern QNAs. The Spanish QNA is no exception and it depends on several specific programs. Procedures for univariate modelling and signal extraction are based on the TRAMO-SEATS programs, see Gómez and Maravall (1996), Caporello and Maravall (2004), and Bógalo (2004). Both programs are critical for the flash estimation of GDP, the standard estimation of QNA, and its seasonal adjustment.

A temporal disaggregation and benchmarking library has been developed (Quilis, 2004a) that may be easily integrated with existing Excel spreadsheets, see Abad and Quilis (2004a)<sup>8</sup>.

The use of econometric libraries (LeSage, 1999) and (Quilis, 2004b), as well as programs for short-term monitoring and cyclical analysis (Fiorentini y Planas, 2002), Abad and Quilis (2004b), are also relevant, specially in the stages of preliminary research, validation and quality control.

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<sup>8</sup> The ECOTRIM program, Barcellan (1994), was instrumental in the consolidation of temporal disaggregation methods in the Spanish QNA.

## APPENDIX A: TENTATIVE DESIGN OF A QSU MODEL

In this Appendix I examine an initial, very tentative proposal to use a Quarterly Supply and Use (QSU) model in the compilation of the Spanish QNA. The initial design assumes:

- Valuation is made at current prices.
- Seasonally adjusted indicators.
- Temporal constraints are not binding.

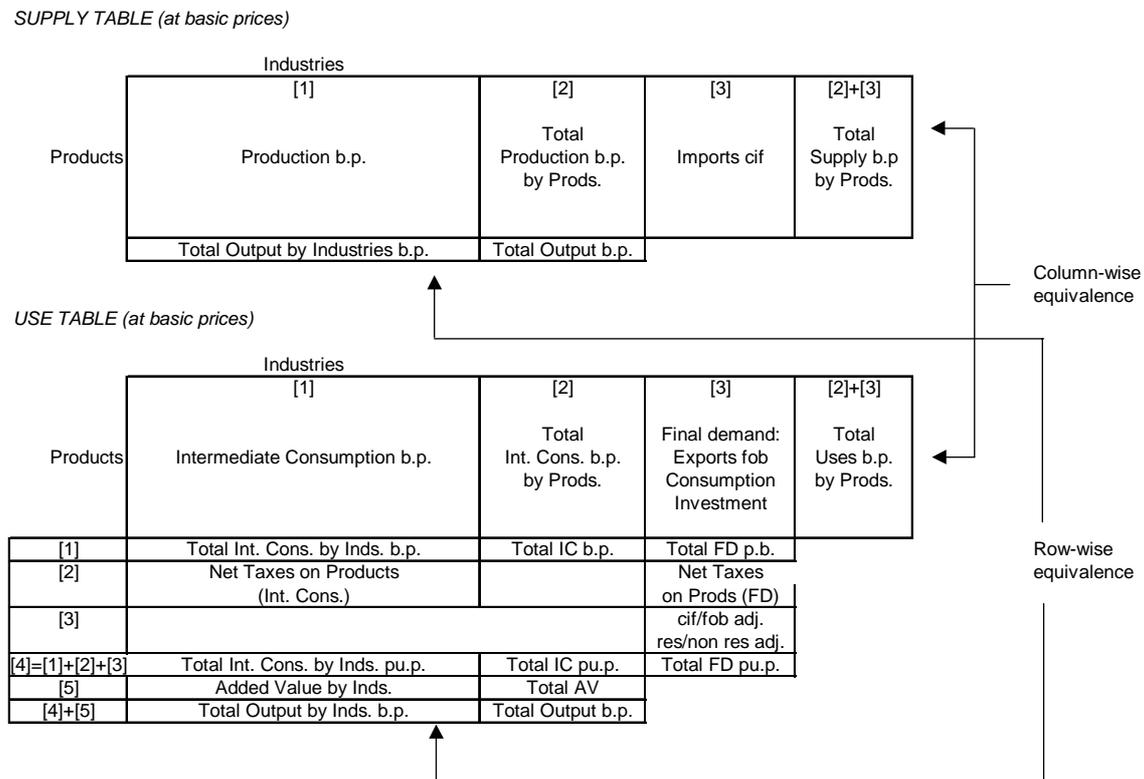
The first assumption avoids the lack of additivity that chain-linked, volumen measures have. The second one is very important in order to ensure consistency with the annual SU tables that provide the structural elements that carry out the estimation process via QSU.

The third assumption stresses that the objective of the QSU model is the estimation of the current quarter. Quarters belonging to years with binding temporal constraints should be estimated using temporal disaggregation procedures.

### • Basic structure

The basic structure of the system and the main restrictions that underlie it are represented in the following diagram:

**Figure 1: QSU structure**



The size of the matrices involved in the system depends on two factors:

- The number (and quality) of the available short-term indicators.
- The operational requirements in order to comply with the QNA dissemination calendar.

Preliminary review suggests that a number of products between 31 and 50 are feasible. Number of industries should be 14 or 17.

- **Estimation**

Short-term quarterly indicators are:

- Extrapolated by means of ARIMA model if required (e.g., incomplete coverage of the reference quarter).
- Adjusted from seasonal variation and calendar effects.

These indicators are the basic element to perform the extrapolation of quarterly data on production, total intermediate consumption, final consumption and gross capital formation. This task may be done using the Chow-Lin-Fernández method. An interesting feature of this approach is that they provide confidence intervals that may be used in the calibration and balancing stages of the QSU model.

Initial estimates of output and intermediate consumption matrices are made by means of the corresponding row coefficients derived from the annual SU model. Net taxes are distributed in a similar way.

Data on exports, imports, total net taxes, cif/fob adjustment and resident-non resident adjustment are obtained directly from basic sources.

- **Calibration**

In order to apply the van der Ploeg method all the elements of the system should be properly gauged. We may consider several cases:

- Calibration related to extrapolated variables (via Chow-Lin-Fernández) may be automatically performed using the output of the underlying models.
- Calibration of matrices may be based on a priori grounds, using a reliability index that measures the degree of uncertainty of each matrix element. These indexes may be translated to variances assuming a specific probability distribution function (e.g., gaussian, uniform, t-Student, etc.)
- Variables that are known with complete precision (e.g. data on external trade) or that we do not want to be transformed should have zero variance (tolerance).
- Some totals may be exactly known but its composition may be uncertain. The best way to consider this case is to assign zero variance to the

aggregate, positive variances to the components and an explicit restriction that links both.

- In some cases, a tight link between variables is necessary, which may be accomplished using specific restrictions like  $y=\theta x$  or  $y-x=a$ .

Detailed calibration is an exercise that should be made only from time to time (e.g., each year) but, if needed, the parameters that define the calibration may be changed if the balancing stage requires it. Finally, from a quality-control point of view, calibration may be used to perform sensitivity analysis.

- **Balancing**

Balancing will be performed using the van der Ploeg (1982) method. The procedure is quite flexible and allows the analyst to incorporate different types of restrictions. The restrictions are included as an additional matrix. Of course, automatically balanced results may be checked, challenged and, eventually, refused and a new cycle of initial (unbalanced) estimation, calibration, and balancing should be performed. In this sense, a valuable piece of information is the variances of the balanced estimates provided by the method.

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